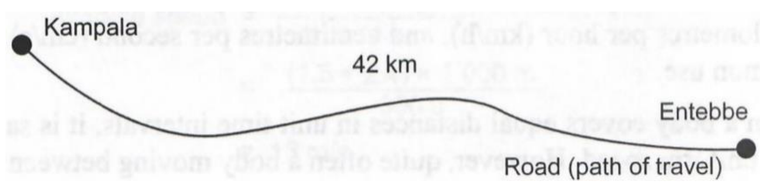
TOPIC: **MECHANICS AND PROPERTIES OF MATTER**
General Objective: The learner should be able to use knowledge of motion and its equations to understand relationship between force, energy and motion.
SUB-TOPIC: Linear motion
SPECIFIC OBJECTIVES
The earner should be able to;
- Define speed and average speed.
- Calculate speed and average speed.
- Define displacement, velocity and acceleration.
- Define uniform velocity and uniform acceleration.
- Draw and interpret velocity graphs for linear motion.
- Use equations of motion to solve numerical problems.
- Use ticker-timer t, find velocity and acceleration.
- Define acceleration due to gravity, g.
- Describe a simple experiment to determine, g.

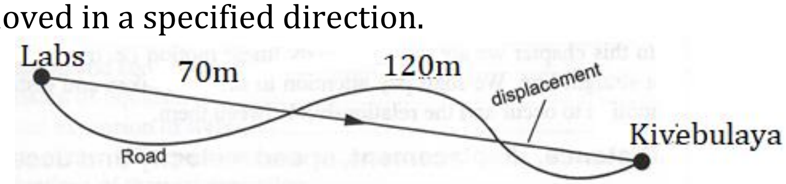
LINEAR MOTION
This involves study of motion of a body in a straight line.

Distance
This is the total length of path travelled by a body.
Or
Is the measurement along the exact path followed by a moving body.

The SI unit of distance is metre (s)



Displacement
This is the distance moved in a specified direction.



If student followed the path from Labs, she would cover a distance of 120m. However, if she moved in a straight line, she would cover a displacement of 70 m.

If the student moved from Kivebulaya house to the Laboratory and back to the house along the path, she would have covered a total distance of 240 m, but her resultant displacement would be 0 m.

So, displacement is a vector because it is described by both magnitude and direction. On the other hand, distance is a scalar.

Speed
Speed is the rate of change of distance with time.

$$\text{Speed} = \frac{\text{distance}}{\text{time taken}}$$

Its SI unit is metre per second (ms^{-1}).
It is a scalar since it is specified by magnitude only.

Uniform speed
A body is said to move with uniform speed if it covers equal distances in unit time intervals.

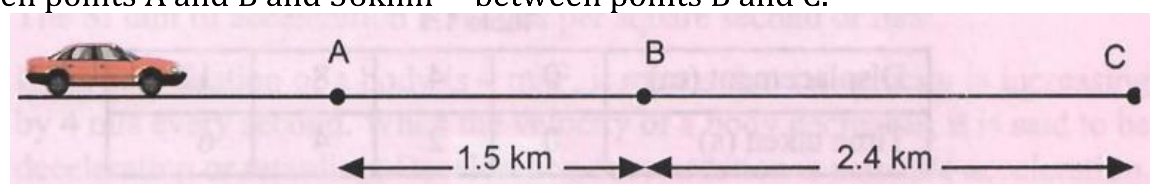
However, quite often a body moving between two points does so with varying speeds. Such a body is said to move with **non-uniform speed**. In such a case the speed between the two points is called average speed.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{total time taken}}$$

Examples

1. What is the speed of a racing car in metres per second if the car covers 360km in 2 hours? ($50ms^{-1}$)

2. A car is moving along a straight road ABC as below maintains an average speed of $90kmh^{-1}$ between points A and B and $36kmh^{-1}$ between points B and C.



Calculate the:

(a) total time taken in seconds by the car between points A and C. (300s)

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{total time between A and B} = \frac{\text{total distance between AB}}{\text{average spedd}}$$

$$= \frac{1.5}{90} \text{ hours} = \frac{1.5}{90} \times 3600 = 60 \text{ s}$$

$$\text{total time between B and C} = \frac{\text{total distance between BC}}{\text{average spedd}}$$

$$= \frac{2.4}{36} \text{ hours} = \frac{2.4}{36} \times 3600 = 240 \text{ s}$$

Total time between A and C $= 60 + 240 = 300$ s.

(b) average speed in metres per second of the car between points ($13ms^{-1}$)

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{(1.5+2.4)\times 1000 \text{ m}}{300 \text{ s}} = \frac{39}{3} = 13 \text{ ms}^{-1}.$$

Velocity

This is the rate of change of displacement with time.

$$\text{Velocity} = \frac{\text{distance moved in a particular direction (displacement)}}{\text{time taken}}$$

The SI unit is metres per second (ms^{-1}).
It is a vector, since it is specified by both magnitude and direction.

In some cases, the velocity of a moving body keeps on changing. In such cases, it is better to consider the average velocity of the body.

A particle is said to move with uniform velocity if its displacement changes by equal amounts in equal time intervals.

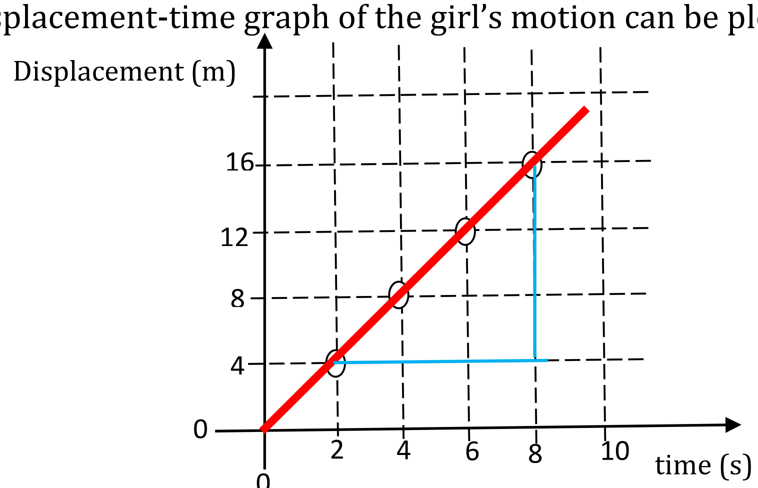
$$\text{Average velocity} = \frac{\text{total displacement}}{\text{time taken}}$$

When the velocity in a particular direction is constant, the velocity is referred to as uniform velocity.

Uniform velocity is the constant rate of change of displacement with time.
For example, the table below shows the displacement of a girl and the corresponding time taken.

Displacement (m)	0	4	8	12
Time taken (s)	0	2	4	6

A displacement-time graph of the girl's motion can be plotted as below:

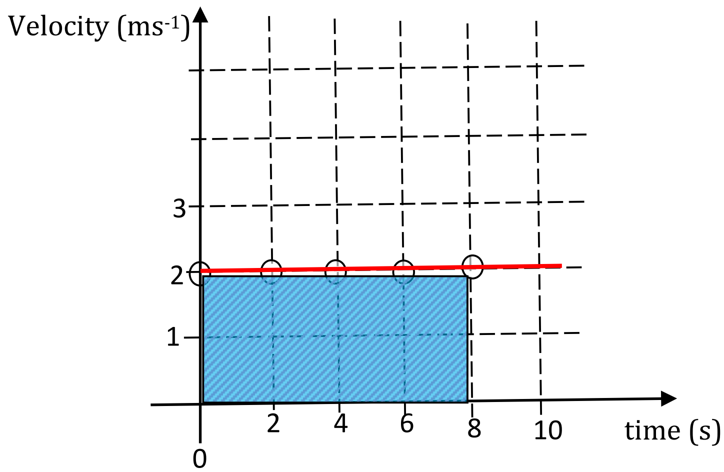


A straight line graph is obtained.
The slope of the graph is the constant velocity at which the girl is moving.

Slope, $v = \frac{\text{change in displacement}}{\text{change in velocity}} = \text{velocity.}$

Slope, $v = \frac{(16-4)\ m}{(8-2)\ s} = \frac{12}{6} = 2\ ms^{-1}$.
The velocity after every two seconds is $2ms^{-1}$, hence its velocity is uniform.
The girls motion can also be represented on a velocity – time graph as below:

Displacement (m)	0	4	8	12
Time taken (s)	0	2	4	6
Velocity (ms^{-1})	0	2	2	2



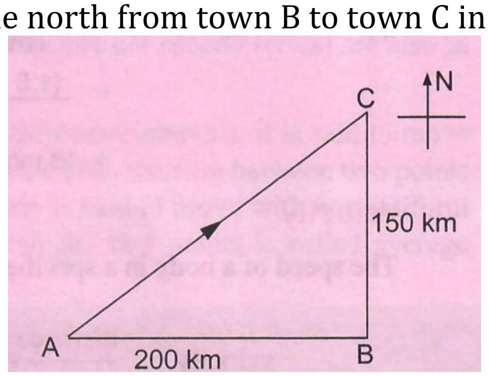
A straight line parallel to the time axis is obtained and this confirms that the girl is moving at a constant velocity.

The **total displacement covered** by the girl in the 8 seconds can obtained as **the area under the velocity –time graph** as follows:

Displacement = area under velocity time graph

Displacement = velocity × time = 2 × 8 = 16 m.

Example
A car travelled from town A to town B 200km east of A in 3hours. The car changed direction and travelled a distance of 150km due north from town B to town C in 2 hours as shown below.



Calculate the average;
(a) speed for the whole journey. (70 kmh^{-1})
(b) velocity for the whole journey. (50 kmh^{-1}) from A to C.

Acceleration
A body is said to be accelerating when its velocity changes.

Definition
This is the rate of change of velocity with time.

$$\text{Acceleration} = \frac{\text{change in velocity of body}}{\text{time taken}}$$

Its SI unit is metres per square second (ms^{-2}).

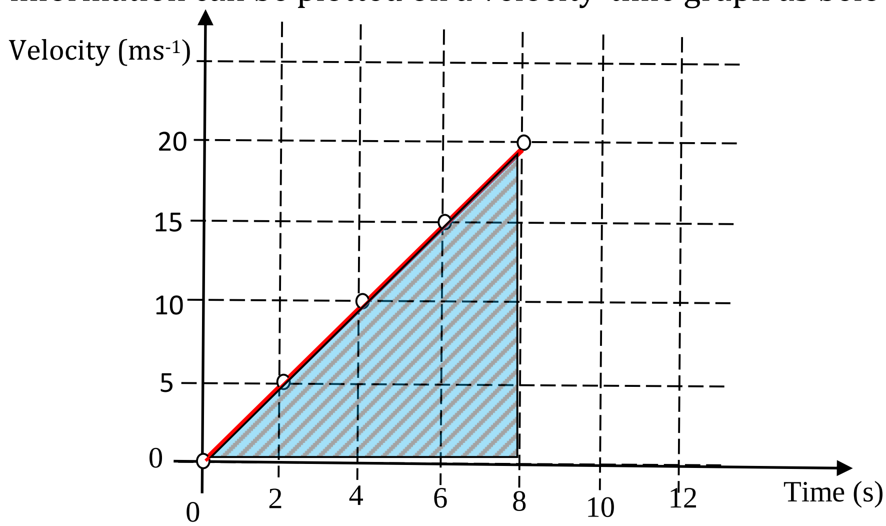
If the acceleration of a body $4ms^{-2}$, it means that its velocity is increasing by $4ms^{-1}$ every second.
NOTE

- Acceleration can be positive or negative. If the acceleration is increasing then it is taken to be positive and if it is decreasing (decelerating or retarding), it is taken to be negative.
- A body moving with **uniform velocity** has **zero acceleration** since there is **no change in velocity.**

When the rate of change of velocity with time is constant, the acceleration is referred as **uniform acceleration**.
Consider a body moving with velocity, v, in time, t, as shown in the table below.

velocity (ms^{-1})	0	5	10	15	20
Time taken (s)	0	2	4	6	8

The information can be plotted on a velocity-time graph as below



A straight line graph is obtained. The slope of the graph is a constant value obtained as below:

$$\text{slope} = \frac{\text{change in velocity}}{\text{change in time}} = \text{acceleration, a}$$

$$\text{slope} = \frac{20-5}{8-2} = \frac{15}{6} = 2.5 \text{ ms}^{-2} = \text{acceleration, a}$$

i.e. the velocity increases by $5ms^{-1}$ for every 2 seconds. Thus, the body is said to be accelerating uniformly at 2.5 ms^{-2}

The **displacement of the body** in the 8 seconds is found as **the area under the velocity-time graph** as follows.

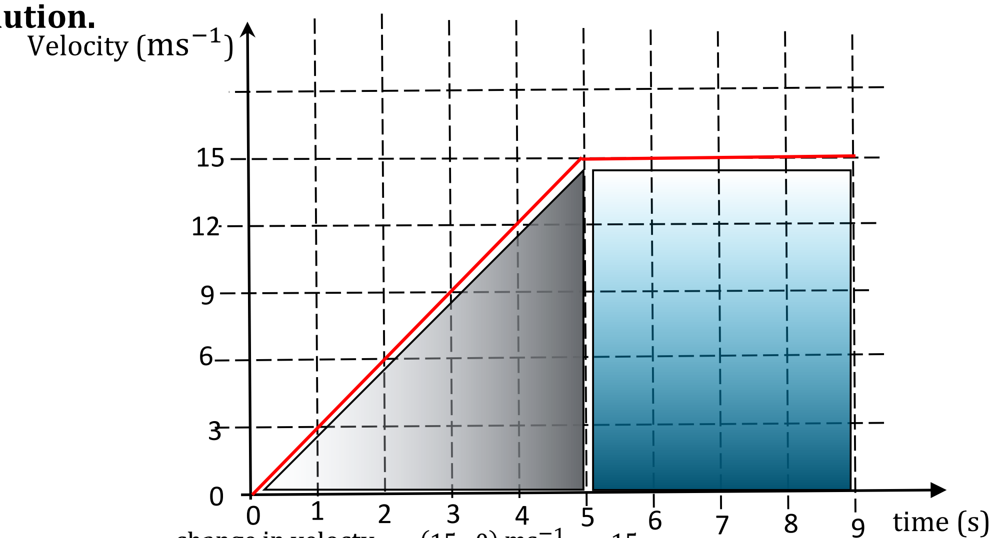
$$\text{Displacement, s} = \frac{1}{2} \times \text{velocity} \times \text{time} = \frac{1}{2} \times 8 \times 20 = 80 \text{ m.}$$

Example:
The table below represents the velocity of a vehicle after a given time.

Velocity (ms^{-1})	0	3	6	9	12	15	15	15	15	15
Time taken (s)	0	1	2	3	4	5	6	7	8	9

(a) Plot a velocity – time graph representing the motion of the vehicle.
(b) Find the slope of the graph in the first 5 seconds of the vehicle's motion.
(c) Use the graph to describe the motion of the vehicle in the 9 seconds.
(d) Use the graph to determine the total displacement of the vehicle in the 9 seconds of its motion.

Solution.
(a)



(b) slope $= \frac{\text{change in velocty}}{\text{change in time}} = \frac{(15-0)\ ms^{-1}}{(5-0)\ s} = \frac{15}{5} = 3\ ms^{-2}$.
(c) In the first 5 seconds, the vehicle accelerates uniformly at 3 ms^{-2} until it attains a velocity of 15 ms^{-1}. It the moves with this velocity constantly for the next 4 seconds.
(d) Displacement = area under the velocity— time graph
Displacement, s = area of the trapezium $= \frac{1}{2} \times 15 \times (9 + 4)$
Displacement, $s = 97.5$ m

Example
A car accelerates from rest to a velocity of $20ms^{-1}$ in 5s. thereafter it decelerates to rest in 8s. Calculate the acceleration of the car.
(a) in the first 5s, ($4ms^{-2}$)

$$\text{Acceleration } = \frac{\text{change in velocity of body}}{\text{time taken}}$$

$$\text{Acceleration } = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

(rest means velocity is zero)

$$\text{Acceleration } = \frac{20-0}{5} = 4ms^{-2}$$

(b) in the next 8s. ($-2.5ms^{-2}$)

$$\text{Acceleration } = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}} = \frac{0-20}{8}$$

$-2.5ms^{-2}$ Or deceleration $= 2.5ms^{-2}$or retardation $= 2.5ms^{-2}$

MOTION –TIME GRAPHS and THEIR INTERPRETATION.

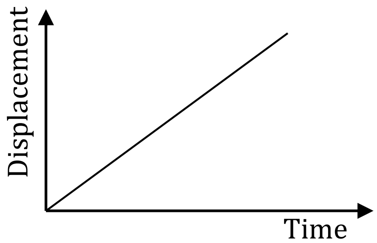
The motion of objects can be represented graphically. During the objects motion, the displacement and velocity of the body usually changes with time. Consideration will be given to:

1. Displacement-time graphs.
2. Velocity-time graphs.

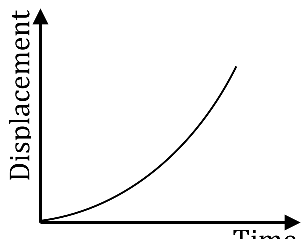
and the information we derive from them.

1. Displacement –Time Graphs

(a) **Moving with constant velocity/uniform**



(b) Accelerating uniformly (non-uniform velocity)

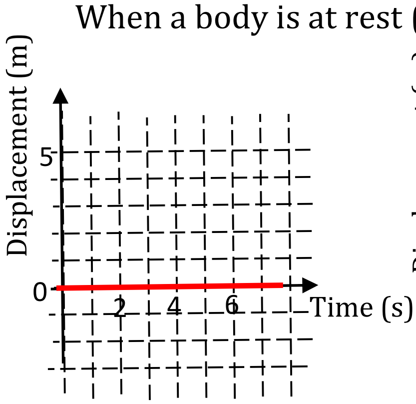


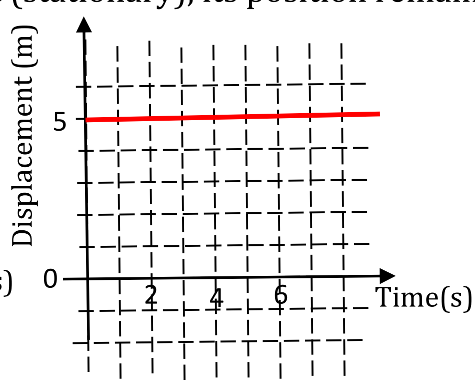
This graph is an example of a stone that drops from rest, the displacement covered in each second is not equal but rather increasing.

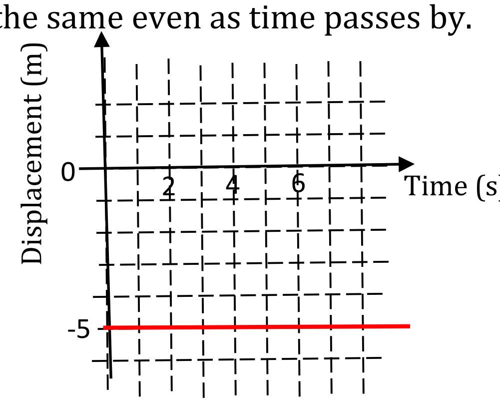
1. **A body at rest**.

(c) Displacement-time graph for a body at rest.

When a body is at rest (stationary), its position remains the same even as time passes by.







(i) The displacement-time graph represents a body that is at rest at its original position.

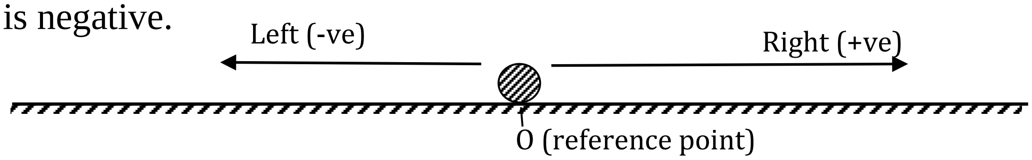
(ii) The displacement-time graph represents a body that is at rest 5 m after or away from its original position.

(iii) The displacement-time graph represents a body that is at rest and 5 m before its original position.

Note: (i) The **gradient of a displacement-time graph** gives the **velocity of the particle**.

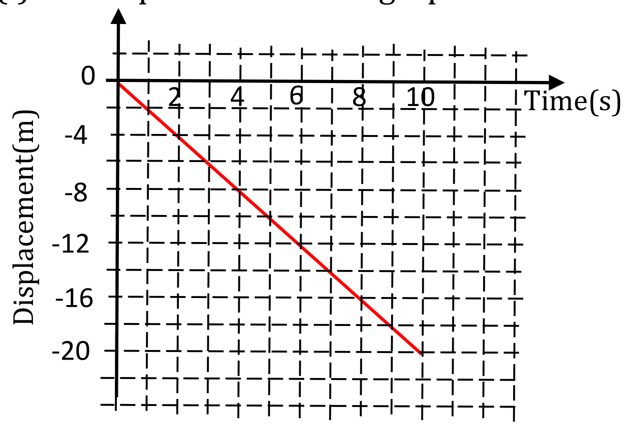
(ii) When the body is stationary the gradient of the graph of the graph is zero and hence the velocity is zero.

A body may be moving to the left or right away from the reference point, O.
The displacement to the right of the reference point is considered to be positive, while that to the left is negative.

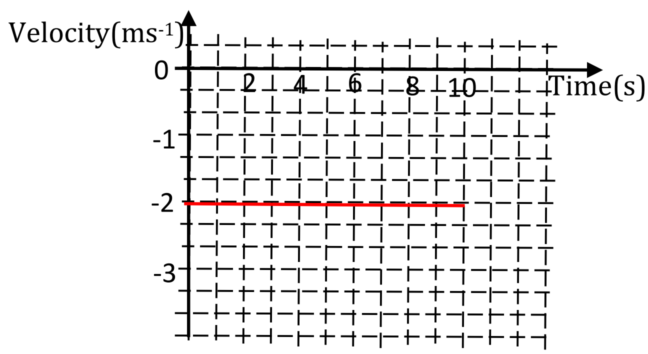


Suppose the body is moving to the left of its reference point, O, graphs are as shown below.

(i) Displacement-time graph



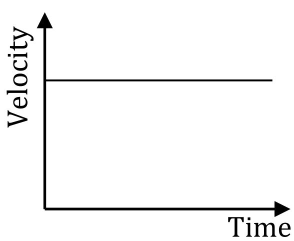
(ii) Velocity-time graph



The slope of the displacement-time graph is $\frac{\text{change in displacement}}{\text{change in time}} = \frac{-20}{10} = -2\text{ms}^{-1}$, the negative sign means the body is moving in the opposite direction.

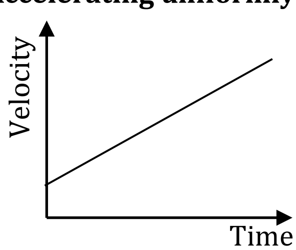
2. Velocity – Time Graphs

(a) **Moving with constant velocity/uniform velocity**



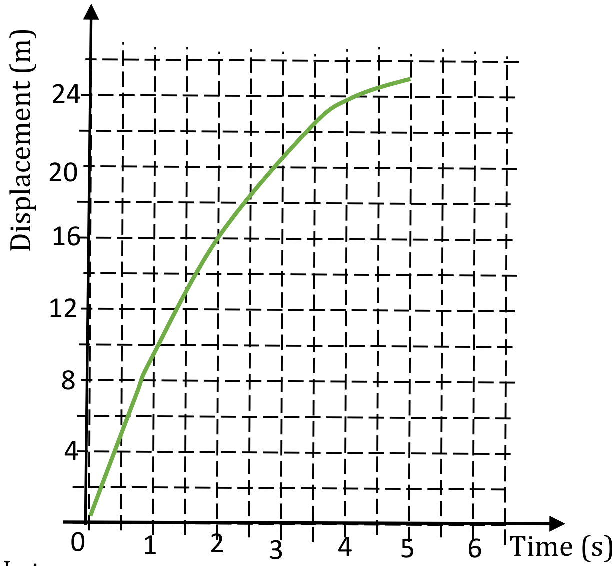
Here, the acceleration is zero Since the velocity is the same i.e. does not increase or decrease.

(b) **Accelerating uniformly**

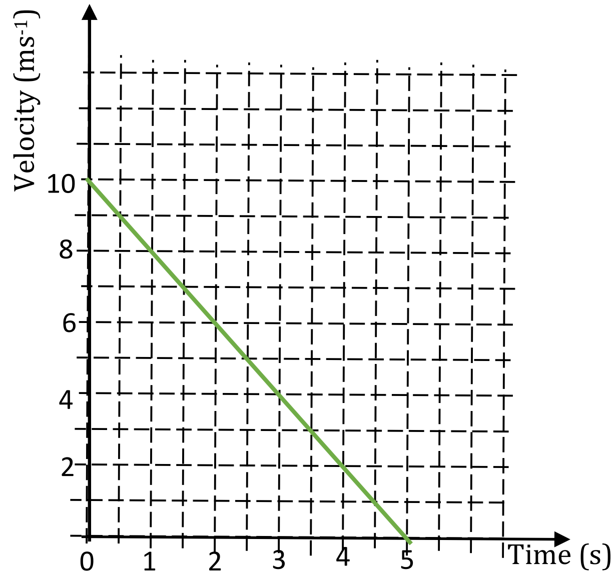


(c) The displacement-time and velocity-time graphs for a body that is decelerating uniformly are as below:

(i) Displacement-time graph



(ii) Velocity-time graph



Note:

(i) The slope of the displacement-time graph is non-uniform but decreases with time.

(ii) The slope of the velocity-time graph is uniform and is obtained as below:

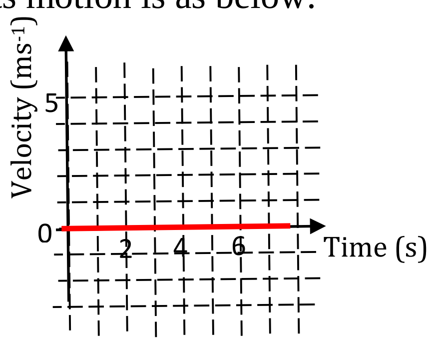
$$\text{slope of velocity} - \text{time graph} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{10-0}{0-5} = -2\ \text{ms}^{-2}.$$

The negative sign means the body is decelerating at a rate of 2 ms^{-2}.

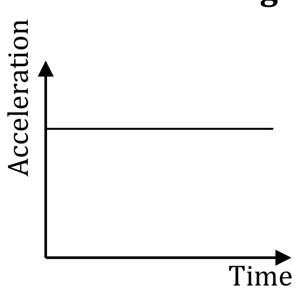
The two graphs above represent motion of a body **decelerating uniformly.**

Velocity-time graph for a body at rest.

When a body is at rest or stationary, its velocity is 0 ms^{-1}. The velocity-time graph representing its motion is as below:



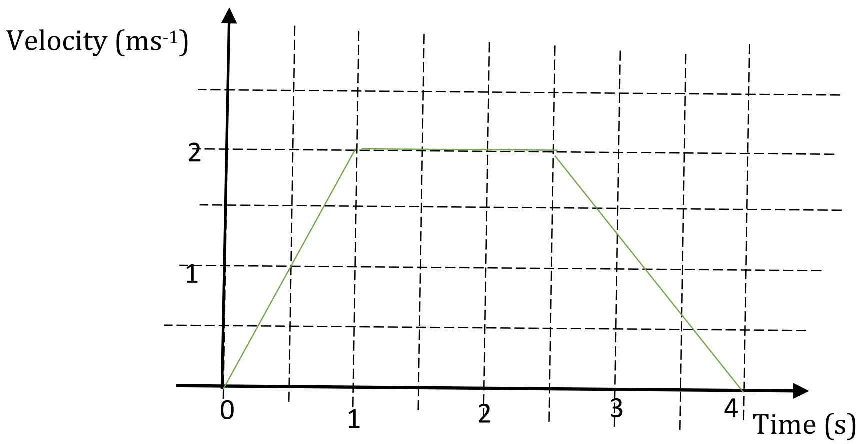
Acceleration-time graph for a body moving with uniform acceleration.



The Area under the velocity-time graph:

- The area under a velocity-time graph is equivalent to displacement of the object.
- The distance covered by object can also be obtained from the velocity-time graph if the direction of the motion is ignored.

Example 1.
The figure below is a velocity-time graph of a car. Use the graph to find
(a) the acceleration of the car.
(b) the deceleration of the car.
(c) the total displacement of the car.



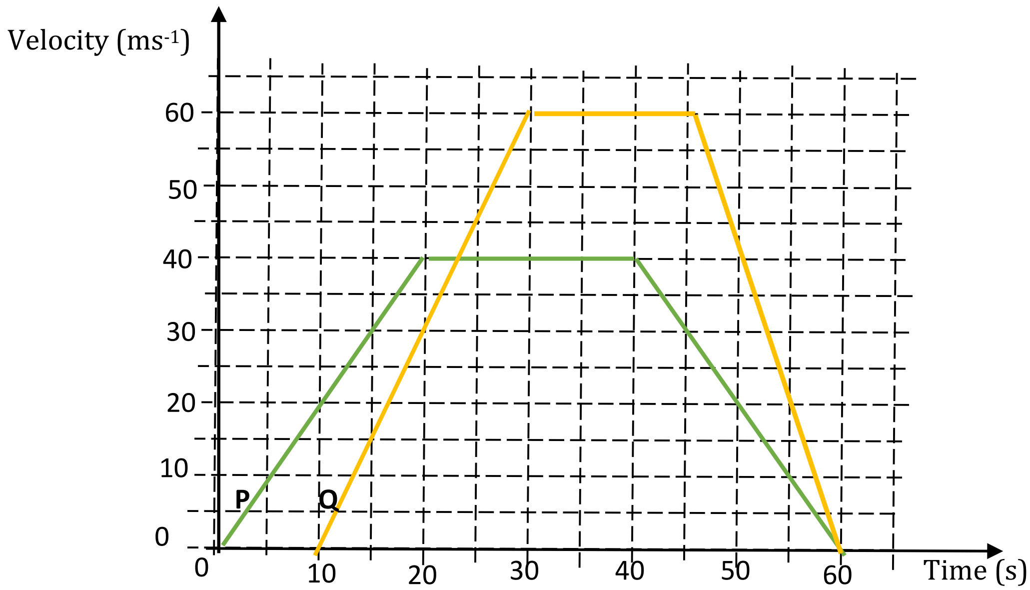
Solution:(a) Acceleration, $a = \frac{\text{change in velocity}}{\text{time}} = \frac{20}{10} = 2\ \text{ms}^{-2}$.

(b) Deceleration $= \frac{\text{change in velocity}}{\text{time}} = \frac{0-20}{40-25} = \frac{-20}{15} = \frac{-4}{3}\ \text{ms}^{-2}$.

The deceleration $= \frac{4}{3}\ \text{ms}^{-2}$.

(c) Total displacement $=$ area under the velocity $-$ time graph
Total displacement $=$ area of trapezium
Total displacement $= \frac{1}{2} \times 20 \times (40 + 15) = 550\text{m}$.

Example 2.
The velocity – time graph below represents the motion of two cars P and Q which start from the same place and move in the same direction. Use the graph to answer the following questions.
(a) Calculate the accelerations of cars P and Q.
(b) Determine how far apart the cars are from each other at the end of their accelerations.
(c) Find the total distance covered by each car.



Solution:

(a) Acceleration of car P:

$a = \frac{v-u}{t} = \frac{40-0}{20-0} = 2\ ms^{-2}$.

Acceleration of car Q:

$a = \frac{v-u}{t} = \frac{60-0}{30-10} = 3\ ms^{-2}$.

(b) Distance moved by car P during its acceleration:

$s_p = \frac{1}{2} \times 20 \times 40 = 400\ m$

Distance moved by car P during its acceleration:

$s_q = \frac{1}{2} \times (30 - 10) \times 60 = \frac{1}{2} \times 20 \times 60 = 600\ m.$

The distance between the cars by the end of their accelerations is $(600 - 400) = 200\ m$.

(c) Total distance moved by car P:

$s_p = \frac{1}{2} \times 40 \times (60 + 20) = 20 \times 80 = 1600\ m.$

Total distance moved by car Q:

$s_p = \frac{1}{2} \times 60 \times (50 + 15) = 30 \times 65 = 1600\ m.$

Example three.

A body is moving at a velocity of $5ms^{-1}$ for 6s. Draw a velocity time graph for the body's motion and use the graph to the distance it covers in 6s.

Example four.

A car starting from rest at P accelerates uniformly for 10 s to a velocity 25 ms^{-1}. It then moves at this constant velocity for 8 s before retarding uniformly for 5 s so as to stop at Q.

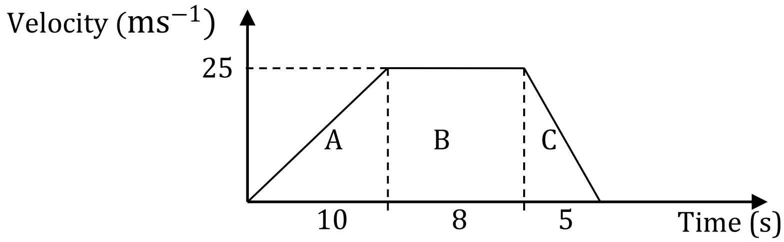
Sketch the velocity-time graph for the car's motion between points P and Q and find

(i) the distance covered during each of the parts of the journey described.

(ii) the acceleration of the car

(iii) the retardation of the car.

Solution



(i) The distance covered during acceleration is the area A

$= \frac{1}{2}$ x 10 x 25 $=$ 125 m

The distance covered at constant speed is the area B

$=$ 8 x 25 $=$ 200 m

The distance covered during retardation is the area C

$= \frac{1}{2}$ x 5 x 25 $=$ 62.5 m

(ii) Acceleration $= \frac{25}{10} = 2.5\ ms^{-2}$

(iii) Retardation $= \frac{25}{5} = 5.0\ ms^{-2}$

Example five.

A car initially at rest accelerates at $2ms^{-2}$ for 10s. It then maintains this new velocity for another 10s before retarding (decelerating) to rest in 5s.

(a) Draw a velocity – time graph for the motion of the car.

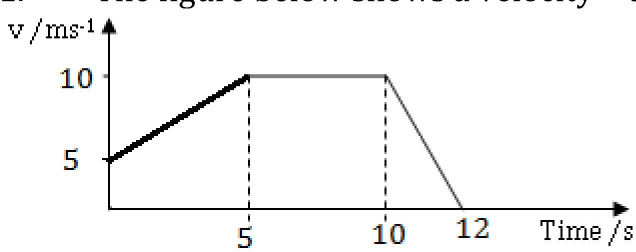
(b) Find the velocity of the car after the first 10s.

(c) Find the total distance covered by the car.

(d) Find the average velocity of the car.

Exercise.

1. The figure below shows a velocity – time graph for the motion of the body.



(a) Describe the motion of the body.

(b) Find the acceleration of the body.

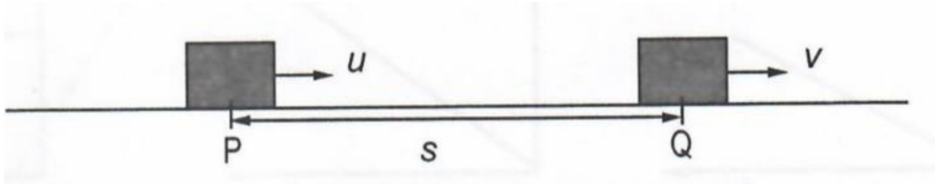
(c) Find the deceleration of the body.

(d) Find the average velocity of the body.

Attempt Exercise 1.1 on pages 14-18

Equations of Uniformly Accelerated Motion.

Suppose a body, originally moving with a velocity **u**ms^{-1} accelerates uniformly at a rate **a** ms^{-2} for **t** seconds such that its final velocity is **v** ms^{-1} after moving through a displacement of **s** metres as shown below,



then;

The first equation of linear motion is obtained as follows:

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

$$a = \frac{v-u}{t}$$

$$\therefore \quad at = v - u$$

$$\therefore \quad \mathbf{v = u + at} \text{ .... } \mathbf{(1)}$$

The second equation of linear motion:

The displacement, **s**, of the particle during this time is given by

Displacement = Average velocity × time

$$\therefore \quad s = \left(\frac{v+u}{2}\right)t$$

but $v = u + at \quad \therefore$

$$s = \left(\frac{u+at+u}{2}\right)t$$

$$s = \frac{2ut + at^2}{2}$$

$$\therefore \quad \mathbf{s = ut + \frac{1}{2}at^2} \text{ .... } \mathbf{(2)}$$

Third equation of linear motion.

Displacement = Average velocity × time

$$\therefore \quad s = \left(\frac{v+u}{2}\right)t$$

From the first equation of motion, $t = \frac{v-u}{a}$

$$\therefore \quad s = \frac{(v+u)}{2}\,\frac{(v-u)}{a} = \frac{v^2-u^2}{2a}$$

$$\therefore \mathbf{v^2 = u^2 + 2as} \text{ .... } (3)$$

The three **equations of uniformly accelerated motion** can be summarized as below:

$$v = u + at \text{ ... ... } (1)$$

$$s = ut + \frac{1}{2}at^2 \text{ ... ... } (2)$$

$$v^2 = u^2 + 2as \text{ ... ... } (3)$$

NB: Retardation

If the velocity of a moving particle decreases with time, then it is said to be retarding (decelerating). In this case, the acceleration is negative.

Examples

1. A particle initially moving with a velocity of 5 m s^{-1} accelerates uniformly at $4ms^{-2}$. Find:
   (i) The velocity of the particle after 8 s.
   (ii) the displacement of the particle after 10 s.
   (iii) the displacement by the time its velocity is 25 m s^{-1}.

   Solution

   (i) Using $v = u + at$, we have
   $v = 5 + (4 \times 8) = 37 \ ms^{-1}$

   (ii) Using $s = ut + \frac{1}{2}at^2$, we have
   $s = (5 \times 10) + (\frac{1}{2} \times 4 \times 10^2) = 50 + 200 = 250$ m

   (iii) Using $v^2 = u^2 + 2as$, we have
   $$s = \frac{v^2 - u^2}{2a} = \frac{25^2 - 5^2}{2 \times 4} = \frac{600}{8} = 75 \text{ m}$$

2. A car, moving with a velocity of 25 ms^{-1} retards uniformly at 2 ms^{-1}. Find:
   (i) the velocity after 8 s.
   (ii) the time it takes to come to rest.
   (iii) the distance covered by the time it comes to rest.

   Solution

   (i) Using $v = u + at$, we have
   $v = 25 + (-2)(8) = 25 - 16 = 9 \ ms^{-1}$

   (ii) Using $v = u + at$, we have
   $0 = 25 + (-2 \times t)$
   $0 = 25 - 2t$
   $\therefore \quad t = 12.5$ s

   (iii) Using $v^2 = u^2 + 2as$, we have
   $0 = 25^2 + 2(-2)s$
   $\therefore \ 4s = 625$
   $$s = \frac{625}{4} = 156.25 \text{ m}$$

3. A car on a straight road accelerates from rest to a speed of 30 ms^{-1} in 5s. It then travels at the same speed for 5minutes and then brakes for 10s in order to come to stop. Calculate the;
   (a) acceleration of the car during the motion. (6 ms^{-2})

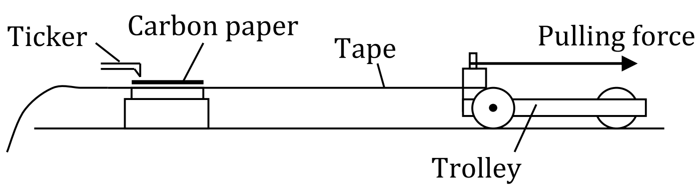
(b) deceleration of the car. (-3 ms^{-2})
(c) total distance travelled. (9000m)

4. The driver of a bus initially travelling at 72kmh^{-1}applies the brakes on seeing a crossing herd of gazelles. The bus comes to rest in 5 seconds. Calculate;
(a) the average retardation of the bus. (-4 ms^{-2})
(b) the distance travelled in this interval. (50m)

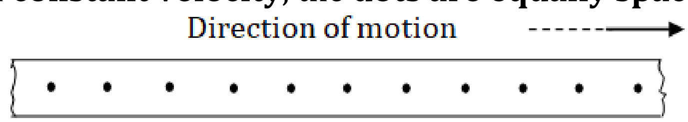
Attempt Exercise 1.2 on pages 21-22 Longhorn Bk 3.

Ticker – tape timer Experiments

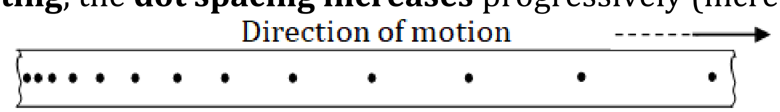
A ticker-timer makes it possible to measure the acceleration of a moving body.
A tape, attached to the body whose motion is being studied, is passed beneath a carbon paper above which is a point that rocks on it at regular time intervals. This way, dots are printed on the tape at regular time intervals.



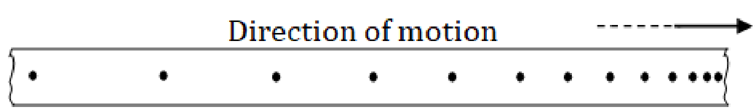
- If the body is moving with **constant velocity**, the **dots are equally spaced** along the tape.



- If the body is **accelerating**, the **dot spacing increases** progressively (increasing velocity).



- If the body is **decelerating/retarding**, the **dot spacing decreases** progressively (decreasing velocity).

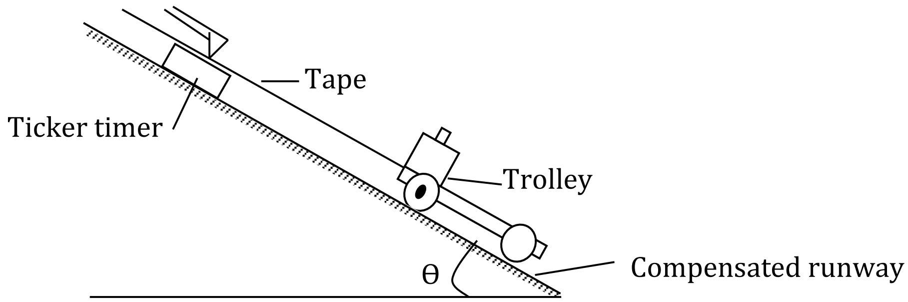


By using different values of the pulling force on the trolley, it can be shown that

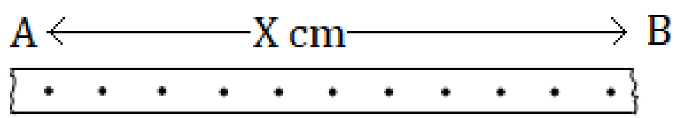
$$\mathbf{a} \propto F$$

Where, F = force,
a = acceleration,
m = mass of the body.
By altering the mass loaded on the trolley, but maintaining the same pulling force, it can be shown that $\mathbf{a} \propto \frac{1}{\mathbf{m}}$

AN EXPERIMENT TO DETERMINE THE UNIFORM VELOCITY OF A BODY USING A TICKER TIMER OF FREQUENCY 50Hz.



- The apparatus is set up as shown above.
- A trolley with a ticker timer attached to it is placed on a horizontal runway (plane).
- The run way is then tilted (inclined) until a point is reached such that when the trolley is given a slight push, the dots printed on the tape by the vibrating ticker timer are equally spaced.
- When the dots are equally spaced, then the trolley is moving with uniform velocity.
- The tape with printed dots is cut out.



- The distance between the dots A and B say 10 dot-spaces apart is measured. Let the distance be x cm.
- Now the calculation is as follows.
- Distance between A and B is x cm $= \frac{x}{100}$ m
- Number of spaces between A and b = 10 dot-spaces

$$f = 50\text{Hz}$$

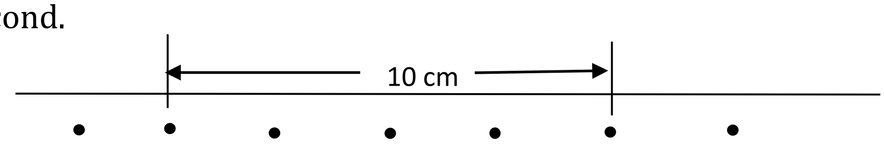
$$T = \frac{1}{f}$$

$$T = \frac{1}{50} = 0.02\text{s}$$

- Time interval between A and B $= nT = 10 \times 0.02 = 0.2s$
- Now using; $\text{Velocity} = \frac{\text{Distance moved in a given direction}}{\text{time taken}}$
- Uniform velocity of the trolley $= \frac{x/100}{0.2} = \frac{x}{20}\text{ms}^{-1}$
- Hence the uniform velocity of body is $\frac{x}{20}\text{ms}^{-1}$

Examples

1. The ticker tape shown below was pulled through a ticker-timer which makes 50 dots per second.

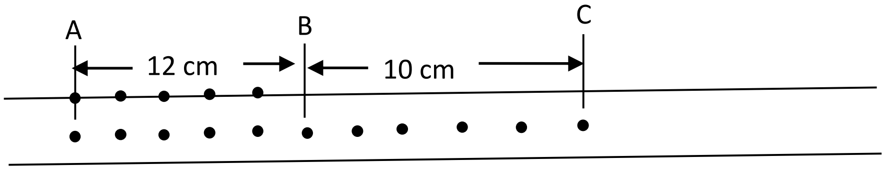


Find the speed at which the tape is being pulled.

Answer:

$$\text{velocity, v} = \frac{\text{distance}}{\text{time}} = \frac{10}{100} \times \frac{50}{4} = 1.25\ \text{ms}^{-1}$$

2. The figure below shows a tape produced by a ticker timer of frequency 50 Hz.

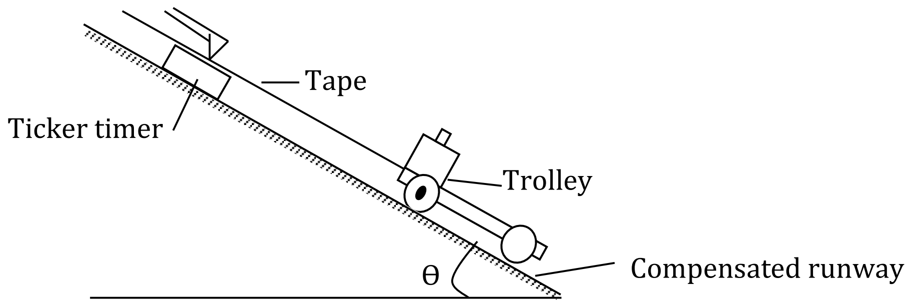


(a) Calculate the time taken to cover distance AB.
(b) The average velocity over the entire motion.
ANSWER:
(a) $\text{time, } t_1 = \text{number of spaces from A to B} \times \text{period}$

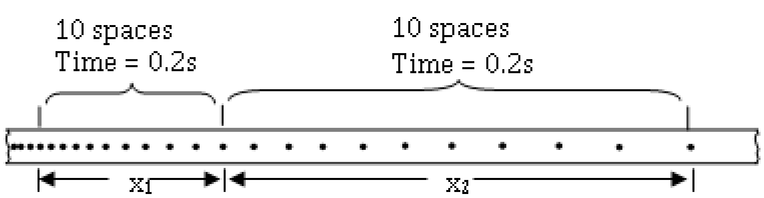
$\text{time, } t_1 = 5 \times \frac{1}{50} = 0.1 \text{ s}$

(b) $\text{average velocity} = \frac{\text{total distance}}{\text{total time taken}} = \frac{(12+10)}{100} \times \frac{50}{10} = 1.1 \text{ ms}^{-1}.$

AN EXPERIMENT TO DETERMINE ACCELERATION OF A BODY USING A TICKER TIMER.



- The apparatus is set up as shown above.
- A ticker timer tape is attached to a trolley.
- The trolley is placed on an inclined plane and the trolley is allowed to run down the plane while pulling the tape through the ticker timer.
- The tape with printed dots is as shown below.



- The distances x_1 and $x_2$between the successive 10 dot spaces are measured.
- The time has a frequency of 50Hz.
- The time taken to print one dot space,

$$f = 50\text{Hz}$$
$$T = \frac{1}{f}$$
$$T = \frac{1}{50} = 0.02\text{s}$$

- Now, the time taken by a 10 dot-space = 10 x 0.02 = 0.2 s
- Average velocity over the distance $x_1 = \frac{x_1}{0.2}$

- And the average velocity over distance $x_2 = \frac{x_2}{0.2}$
- Hence, change in velocity in 0.2 s $= \frac{x_2}{0.2} - \frac{x_1}{0.2} = \frac{x_2 - x_1}{0.2}$
- Now, the acceleration $= \frac{\text{Change in velocity}}{\text{Time}} = \frac{\frac{x_2 - x_1}{0.2}}{0.2}$

$$\therefore \text{Acceleration} = \frac{x_2 - x_1}{(0.2)^2}$$

Examples

1. The figure below shows a tape produced by a ticker – timer operating at a frequency of 50 Hz.

If the trolley was accelerating.

(a) In which direction was it moving?

ANS: B to A, because the dot spacing increase as one moves from A towards B.

(b) Calculate the acceleration of the trolley that pulled the paper tape through the ticker-tape timer.

ANS:

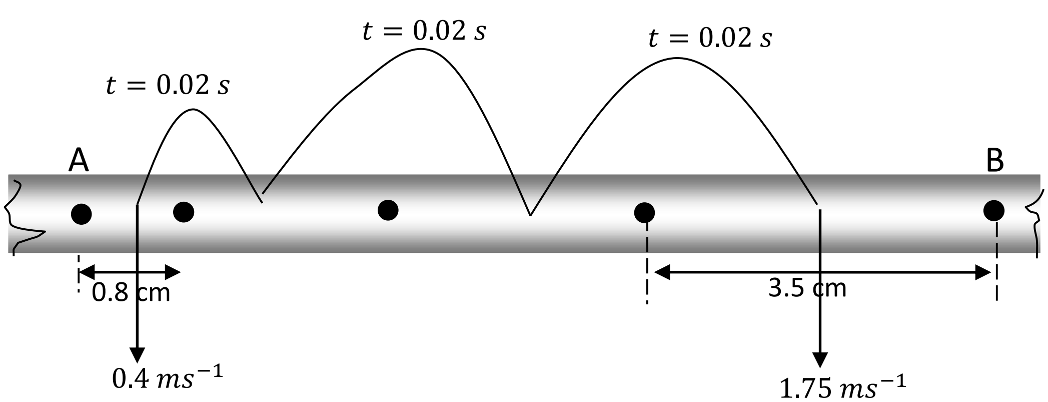
The interval between dots is increasing from A to B, hence the velocity is increasing in the same direction, i.e. the trolley is accelerating. To determine the acceleration, we need to obtain the average initial and final velocities. These are the velocities between the first two dots and the last two dots respectively.

Frequency, = 50 Hz $\therefore$ period, $T = \frac{1}{f} = \frac{1}{50} = 0.02$ s, the time interval between two successive dots.

Initial average velocity between first two dots, $u = \frac{x_1}{T} = \frac{0.008}{0.02} = 0.4\ ms^{-1}$.

Final average velocity between last two dots, $v = \frac{x_2}{T} = \frac{0.035}{0.02} = 1.75\ ms^{-1}$.

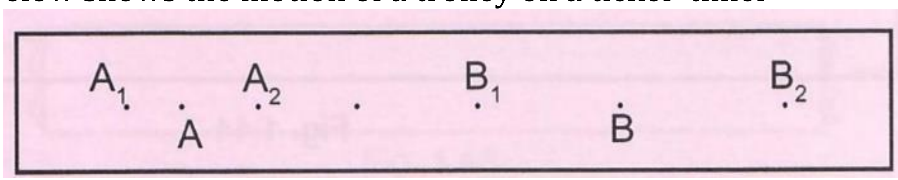
Note: the average velocity between any two dots is equal to the velocity midway between the two dots.



Though there are 4 time intervals between the dots A and B, there are 3 time intervals between the instances of average initial and final velocities (the acceleration period). Thus,

$$\text{Total time taken, t} = \text{period} \times (\text{number of spaces between all dots} - 1)$$
$$\text{Total time taken, t} = \frac{1}{f} \times (n-1)\text{spaces}$$
$$\text{Total time taken, t} = 0.02 \times (4-1) = 0.02 \times 3 = 0.06 \text{ s}$$
$$\text{acceleration, a} = \frac{v-u}{t} = \frac{1.75-0.4}{0.06} = 22.5 \text{ ms}^{-2}$$

2. The figure below shows the motion of a trolley on a ticker-timer

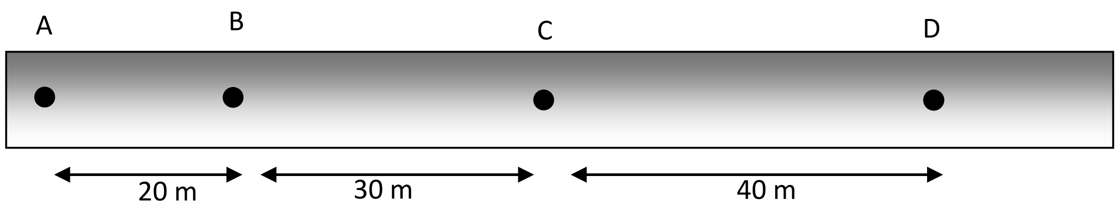


(a) Find the velocity at points A and B. ($30cms^{-1}$ and $70cms^{-1}$)
(b) Calculate the acceleration between points A and B. ($500cms^{-2}$or $5.0ms^{-2}$)

3. In a ticker-timer experiment the distance occupied by a 6-dot space on the tape is 5.1 cm, while the adjacent 6-dot space occupies 6.3 cm. find the acceleration of the body to which the tape is attached, if the ticker frequency is 50 Hz.
Solution
Time taken by 6-dot space $= 6 \text{ x} \frac{1}{50} = 0.12$ s

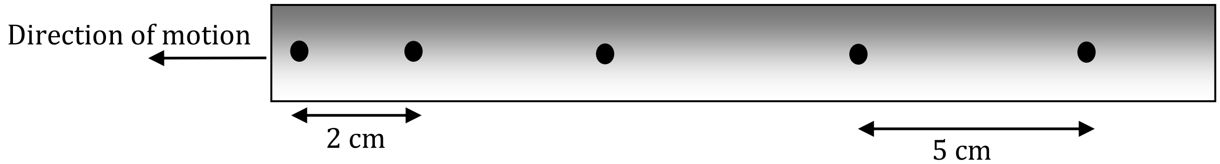
$$\therefore \text{ Acceleration } = \frac{6.3-5.1}{(0.12 \text{ x } 0.12)} = \frac{1.2}{0.0144} = 83.3 \text{ cm s}^{-1}$$

4. Oil was leaking from a car as it travelled along a road. One oil drop fell on the road after every 2 seconds. The figure below shows the pattern formed by the drops on the road. Calculate the acceleration of the car.



Solution.
In terms of time, instances B and C are midway between the time intervals AC and BD respectively.
Time interval between any two drops = 2s.
$$\text{Velocity at point B} = \frac{\text{displacement AC}}{\text{time taken}} = \frac{50 \text{ m}}{4 \text{ s}} = 12.5 \text{ ms}^{-1}$$
$$\text{Velocity at point C} = \frac{\text{displacementBD}}{\text{time taken}} = \frac{70 \text{ m}}{4 \text{ s}} = 17.5 \text{ ms}^{-1}$$
$$\text{Acceleration between B and C} = \frac{V-U}{t} = \frac{17.5-12.5}{2 \text{ s}} = \frac{5}{2} = 2.5 \text{ ms}^{-2}.$$

5. The figure below shows dots produced on a tape pulled through a ticker – timer by a moving body.



The frequency of the ticker timer is 50 Hz. Calculate the acceleration of the body.

Solution:

time interval between two sucessive, $\mathrm{T} = \frac{1}{\mathrm{f}} = \frac{1}{50} = 0.02$ s.

$\mathrm{x}_2 = 5\ \mathrm{cm} = 0.05\ \mathrm{m}$, $\quad \mathrm{x}_1 = 2\ \mathrm{cm} = 0.02\ \mathrm{m}$

Average final velocity, $\mathrm{v} = \frac{\mathrm{x}_1}{\mathrm{T}} = \frac{0.05}{0.02} = 2.5\ \mathrm{ms}^{-1}$

Average initial velocity, $\mathrm{u} = \frac{\mathrm{x}_2}{\mathrm{T}} = \frac{0.02}{0.02} = 1.0\ \mathrm{ms}^{-1}$

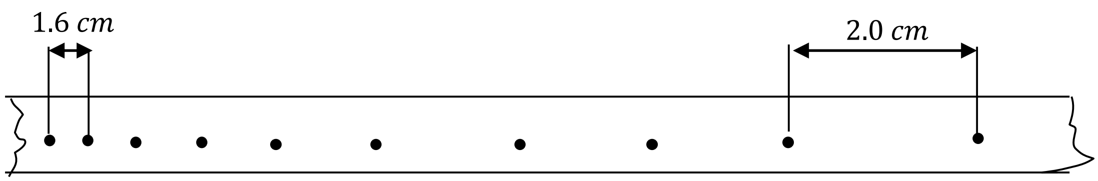
Total time taken between the average initial and final velocities:

$\mathrm{t} = \text{period} \times (\mathrm{n} - 1)\text{spaces}$

$\mathrm{t} = 0.02 \times (4 - 1)\text{spaces} = 0.06$ s

Acceleration, $\mathrm{a} = \frac{\mathrm{v}-\mathrm{u}}{\mathrm{t}} = \frac{2.5-1.0}{0.06} = \frac{1.5}{0.06} = 25\ \mathrm{ms}^{-2}$

6. The figure below figure below shows a section of a tape used to study the motion of a body. The ticker timer used has a frequency of 50 Hz.



Determine the acceleration of the body.

ANSWER:

time interval between two dots = period, $\mathrm{T} = \frac{1}{\mathrm{f}} = \frac{1}{50} = 0.02$ s

initial velocity, $\mathrm{u} = \frac{\mathrm{x}_1}{\mathrm{T}} = \frac{0.016}{0.02} = 0.8\ \mathrm{ms}^{-1}$

final velocity, $\mathrm{v} = \frac{\mathrm{x}_2}{\mathrm{T}} = \frac{0.020}{0.02} = 1.0\ \mathrm{ms}^{-1}$

Total time taken between the average initial and final velocities:

$\mathrm{t} = \text{period} \times (\mathrm{n} - 1)\text{spaces}$

$\mathrm{t} = 0.02 \times (9 - 1)\text{spaces} = 0.16$ s

Acceleration, $\mathrm{a} = \frac{\mathrm{v}-\mathrm{u}}{\mathrm{t}} = \frac{1.0-0.8}{0.16} = \frac{0.2}{0.16} = 1.25\ \mathrm{ms}^{-2}$

7. 2012 p1 no.41

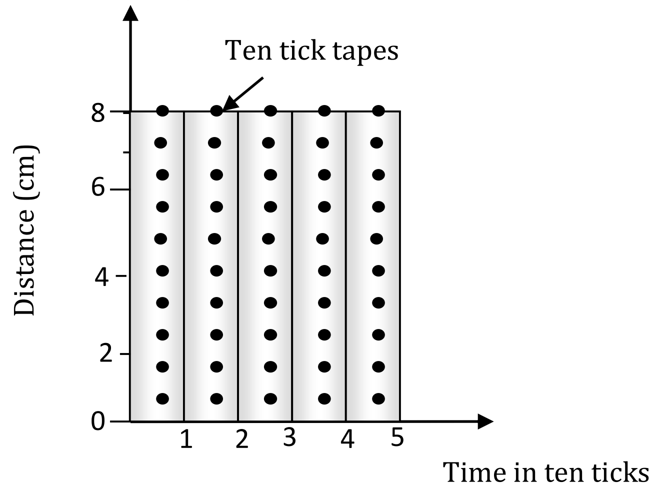
   Leave 15 lines

8. 2014 p1 no.40

   Leave 15 lines

TAPE CHARTS
Tape charts are made by sticking successive strips of tape, usually tentick lengths, side by side.

1. **Tape chart representing Uniform velocity**:

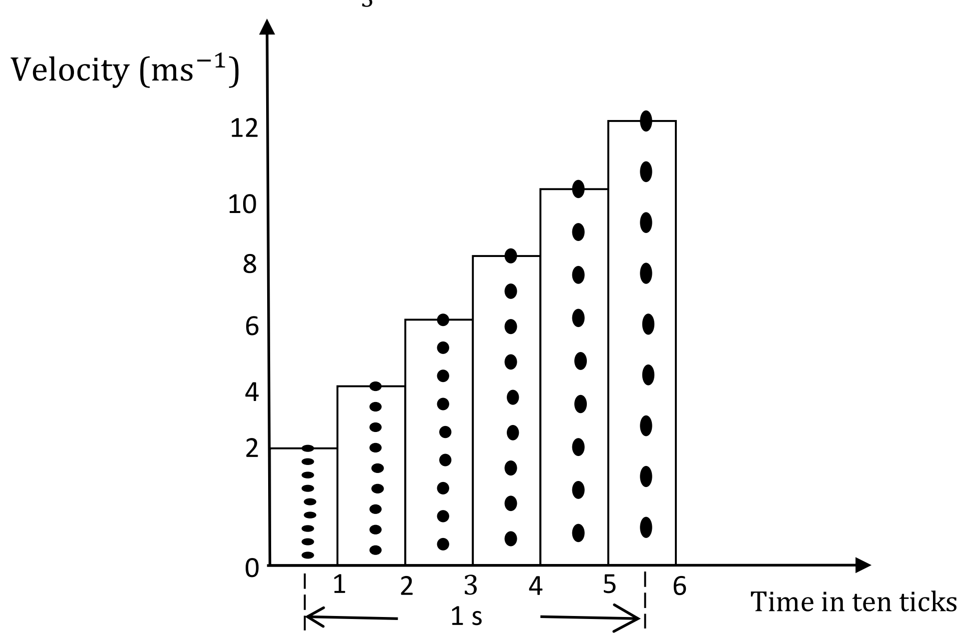


The chart represents motion of a body moving with uniform velocity since equal distance has been moved in each ten tick interval.

$$\text{Velocity} = \frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{\text{period} \times \text{no. of spaces}}$$

$$\text{Velocity} = \frac{8}{\frac{1}{50} \times 10} = 40 \text{ cms}^{-1}$$

2. **Tape chart representing uniform acceleration**.
If the difference in length of tentick tapes is the same, then the tape chart represents motion of a body moving with uniform acceleration (i.e. the speed increased by the same amount in every ten tick interval of $\frac{1}{5}$ s).



The speed during the first tentick is $\mathbf{2\ cm/\frac{1}{5}s}$ or $\mathbf{10\ cms^{-1}}$. During the sixth tentick, it is $\mathbf{12\ cm/\frac{1}{5}s}$ or $\mathbf{60\ cms^{-1}}$. Therefore during this interval of 5 tenticks i.e 1 second ($\mathbf{5 \times 0.2 = 1.0\ s}$), the change of speed is $(\mathbf{60 - 10})\ \mathbf{cms^{-1}} = \mathbf{50\ cms^{-1}}$.

$$\text{acceleration} = \frac{\text{change of speed}}{\text{time taken}} = \frac{50\ \text{cms}^{-1}}{1\ \text{s}} = 50\ \text{cms}^{-2}.$$

Attempt Exercise 1.4 on pages 35-36

Motion under Gravity

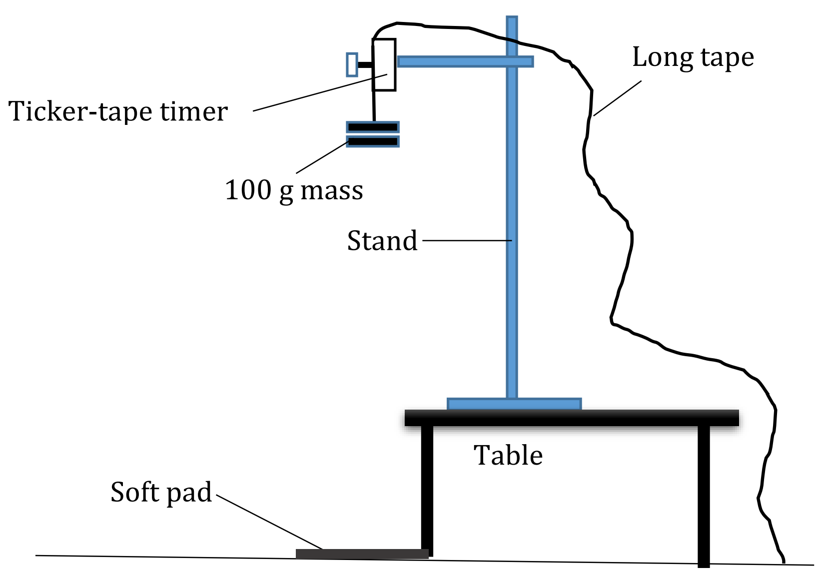
Anybody left to fall freely accelerates at a rate g m s^{-2} towards the centre of the Earth. So, if the body is moving upwards, it retards at this rate, in which case the acceleration is –g. i.e. if the upward direction is taken to be positive, the gravitational acceleration is negative (because it is a retardation).

Definition:
Acceleration due to gravity is the rate of change of velocity of a body falling freely under the influence of the earth's pull on it.
Or
Acceleration due to gravity is the acceleration due to the pull of the earth on the objects.

Experiments to determine acceleration due to gravity:

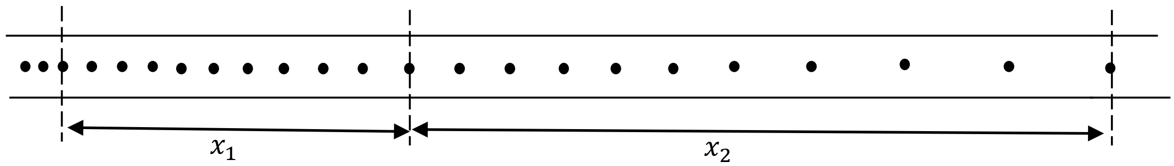
1. Experiment to determine acceleration due to gravity using a ticker-tape timer.

A ticker-tape timer is clamped as shown in the diagram at a height of about 2m above the ground.



A 100 g mass attached to one end of the tape passing through a ticker-tape timer is released to fall freely under gravity and at the same time, the ticker-tape timer is switched on.

The acceleration due to gravity is analyzed from the tape obtained as below:



The first few dots are ignored because they are too close to be distinguished from each other. The distances x_1 and x_2 occupied by successive 10 dot-spaces are measured.

The time taken by a 10 dot-space = $10 \times 0.02 = 0.2$ s

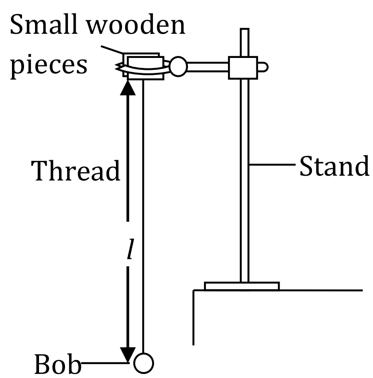
$\therefore$ average velocity over the distance $x_1 = \frac{x_1}{0.2\ s}$

similarly, the average velocity over distance $x_2 = \frac{x_2}{0.2\ s}$

$$\text{hence, change in velocity in 0.2 s} = \frac{x_2}{0.2} - \frac{x_1}{0.2} = \frac{x_2 - x_1}{0.2}$$

$$\text{Now, the acceleration due to gravity. g} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{x_2 - x_1}{(0.2s)^2}$$

An experiment to determine acceleration due to gravity using a simple pendulum.



The apparatus is assembled as shown in the diagram.

Starting, with a string length of $l = 100$ cm, the pendulum bob is displaced through a small angle, θ and then released to oscillate freely.

A stop watch is used to time 20 oscillations of the pendulum and the time taken is recorded as t second.

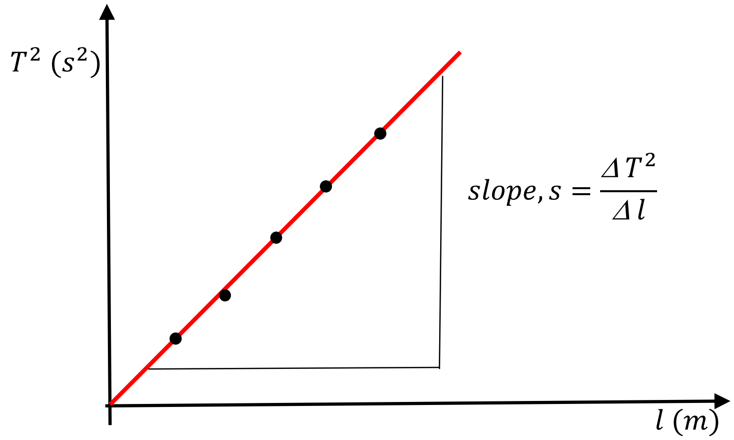
The time, T, taken for one oscillation is calculated as $T = \frac{t}{20}$ s.

The experiment is repeated for at least five different string lengths, that is $l =$ 90cm, 80 cm, 70 cm, 60 cm, 50 cm and 40 cm respectively.

The results are recorded in a suitable table including values of T^2, as below.

l(m)	t(s)	T(s)	$T^2(s^2)$
1.000			
0.900			
0.800			
0.700			
0.600			
0.500			

A graph of T^2 against l is plotted as below.

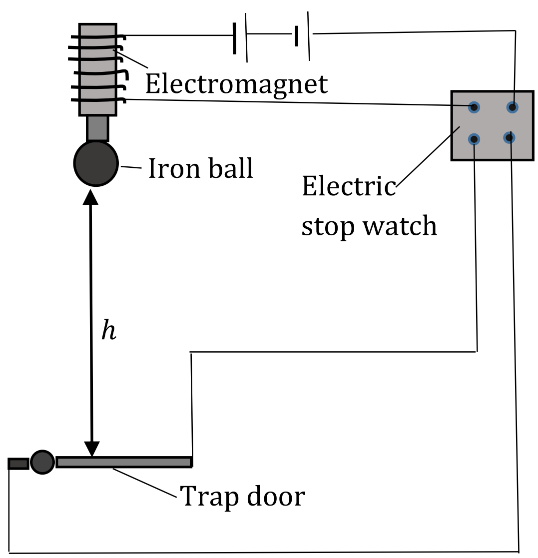


The acceleration due to gravity, g is calculated from the formula

$$g = \frac{4\pi^2}{s}.$$

Experimental results show that the average value of acceleration due to gravity, g is 9.8 ms^{-2}. For purposes of easing calculations, g is approximated to 10 ms^{-2}.

Experiment: To determine acceleration due to gravity using an electromagnet.



The apparatus is set up as shown in the diagram.
When the switch is closed, the electromagnet holds the iron ball in position.
When the switch is opened, the ball falls freely and the clock starts to count at the same time.
The stop clock stops immediately the iron ball hits the trap door and breaks the circuit.
The time recorded, t is the time taken by the iron ball to fall through the distance, h.
The acceleration due to gravity, g is calculated from the equation

$$h = \frac{1}{2}gt^2, \left(\text{i.e. } g = \frac{2h}{t^2}\right).$$

The equations of motion under gravity are:
The equations of motion for a body moving under the influence of gravitational force are.

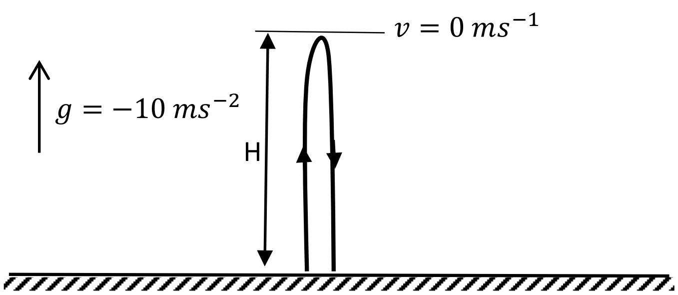
$$\mathbf{v = u + gt}$$

$$\mathbf{h = ut + \frac{1}{2}gt^2}$$

$$\mathbf{v^2 = u^2 + 2gh}$$

- If the body is moving vertically upwards against the force of gravity, it will be decelerating at $g = -10\ \text{ms}^{-2}$.
- If the body is moving vertically downwards in the direction of the force of gravity, it will be accelerating at $g = 10\ \text{ms}^{-2}$.
- All displacements above the point of projection are considered as positive, while those below the point of projection are negative.

Key terms:



1. **Maximum/greatest height, H,**
   This the greatest vertical displacement of the object from the point of projection.

2. **Trajectory.**
   This is the path described by a body in flight or motion.

3. **Time of flight**, T.
   This the total time taken by an object to move from its point of projection and back.
   Note:
   1. At maximum height, the velocity of the object is zero, since the object is momentarily at rest. (i.e. $v = 0\ \text{ms}^{-2}$) .
   2. The time taken to reach maximum height, t is half the time of flight, T.
      i.e. $t = \frac{1}{2}T$.

Examples

1. A particle is projected vertically upwards with a velocity of 20 m s^{-1}. Find:
   (i) the greatest height the particle attains.
   (ii) the time taken to attain the greatest height.
   (iii) the velocity and direction of motion after 3 s of motion.
   (iv) the height 3 s after projection.
   (v) the time of flight.
   [Take g to be 10 ms^{-2}]

   Solution

(i) **At the maximum height, the velocity of the particle is zero**.

Let h $=$ greatest height

Then, using $\mathbf{v^2 = u^2 + 2gh}$, , we have

$0 = 20^2 + 2 \times (-10)$

$\therefore \quad h = 20$ m

(ii) Using $\mathbf{v = u + gt}$, where t is the time required to reach the greatest height, we have

$0 = 20 + (-10)t$

$\therefore t = 2$ s

(iii) Using $v = u + gt$, where v is the velocity after 3 s, we have

$$v = 20 - 10 \times 3 = -10 \text{ ms}^{-1}$$

Since we chose the upward direction to be positive, the negative sign implies that the particle is moving downwards.

(iv) Using $h = ut + \frac{1}{2}gt^2$, we have

$$h = 20 \times 3 + \frac{1}{2} \times (-10) \times 3^2 = 15 \text{ m}$$

(v) The total displacement of the stone during the time of flight is zero.

From $h = ut - \frac{1}{3}gt^2$

$0 = 20T - \frac{1}{2} \times 10 \times T^2$

$0 = 5T(4 - T)$

either $T = 0$ or $T = 4$

The time of flight, $T = 4$ s.

2. A stone released from the top of a tree hits the ground after 3 s. Find:
   (i) the height of the tree.
   (ii) the velocity with which it hits the ground.

Solution

(i) We may take the downward direction as positive. So, the acceleration is $g = 10 \text{ ms}^{-2}$. Using $h = ut + \frac{1}{2}gt^2$, where $u = 0 \text{ ms}^{-1}$, we have

$h = 0 + \frac{1}{2} \times 10 \times 3^2 = 45$ m

The tree is 45 m tall.

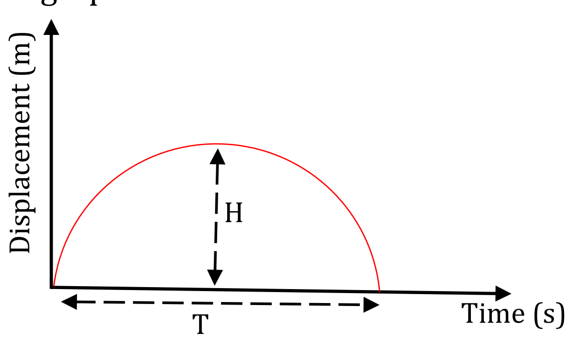
(ii) Using $v = u + gt$, we have

$v = 0 + 10 \times 3 = 30 \text{ ms}^{-1}$

The stone hits the ground with a velocity of 30 ms^{-1}.

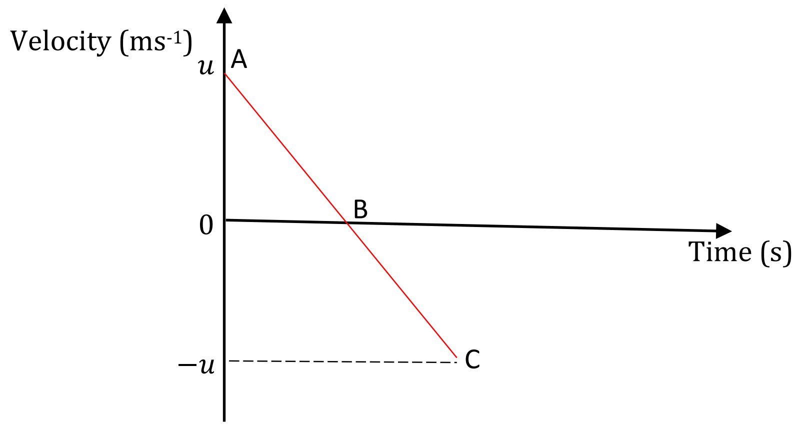
Graphs of motion under gravity.

1. Displacement –Time graph.



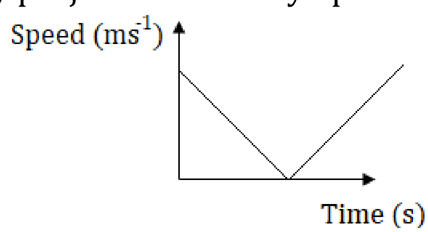
The object decelerates on its upward journey until it reaches maximum height, H when it is momentarily at rest. It the changes direction as it accelerates downwards.

2. Velocity-time graph.



At A, the body is projected vertically upwards with an initial velocity of u ms^{-1}.
It then decelerates uniformly at a rate of 10 ms^{-2} until it comes to momentarily at rest at B. It then changes direction and accelerates downwards until it comes back to its original position of projection.

3. Speed-time graph for a body projected vertically upwards



Effects of variation in force of gravity.
The gravitational force varies from planet to planet. Therefore,
1. different planets have different accelerations due to gravity.
2. the weight of a body varies from one planet to the other (i.e. W = mg).
3. the mass of the body remains constant on all planets.

Examples
1. A particle is projected vertically upwards with a velocity of 20 m s^{-1}. Find:
 (i) the greatest height the particle attains
 (ii) the time taken to attain the greatest height
 (iii) the velocity and direction of motion after 3 s of motion
 (iv) the height 3 s after projection
 [Take g to be 10 ms^{-2}]

Solution
(i) At the highest point the velocity of the particle is zero
Let h = greatest height
Then, using $v^2 = u^2 - 2gh$, we have
$0 = 20^2 - 2 \times 10h$
$\therefore \quad h = \underline{20 \text{ m}}$

(ii) Using $v = u - gt$, where t is the time required, we have

$0 = 20 - 10t$

$\therefore \quad t = 2\ s$

(iii) Using $v = u - gt$, where v is the velocity after 3 s, we have

$v = 20 - 10 \times 3 = -10\ m\ s^{-1}$

Since we chose the upward direction to be positive, the negative sign implies that the particle is moving downwards.

(iv) Using $h = ut - \frac{1}{2}gt^2$, we have

$h = 20 \times 3 - \frac{1}{2} \times 10 \times 3^2 = \underline{\underline{15\ m}}$

2. A stone is released from the top of a tree and hits the ground after 3 s. Find:
(i) the height of the tree
(ii) the velocity with which it hits the ground

Solution

(i) We may take the downward direction as positive. So, the acceleration is $g = 10\ m\ s^{-2}$.

Using $h = ut + \frac{1}{2}gt^2$, where u = 0, we have

$h = 0 + \frac{1}{2} \times 10 \times 3^2 = \underline{\underline{45\ m}}$

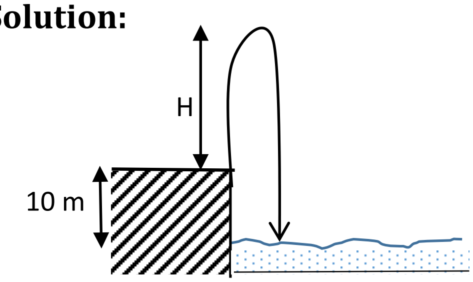
(ii) Using $v = u + gt$, we have

$v = 0 + 10 \times 3 = \underline{\underline{30\ m\ s^{-1}}}$

3. A body is thrown vertically upwards with an initial velocity of $20ms^{-1}$. Given that the gravitational pull $g = 10ms^{-2}$, find
(i) the time the body takes to reach the maximum height.,(2s)
(ii) the maximum height reached above the starting point.(20m)
(iii) the total time of flight.(4s)

4. A particle is projected vertically upwards with a velocity of 20 $ms^{-1}$from the edge of a cliff that is 10 m above the sea level. Find
(i) the maximum height reached by the particle.
(ii) the velocity at which the particle hits the water.
(iii) the total time taken for the particle to hit the sea.
(acceleration due to gravity, $g = 10\ ms^{-2}$)

Solution:



$v^2 = u^2 + 2gh$

$0 = 20^2 + 2(-10)H$

Maximum height, $H = \frac{400}{20} = 20$ m.

(ii) The resultant displacement of the particle by the time it hits the water is 10 m below the edge of the cliff

i.e. $h = -10$ m

Applying equation : $v^2 = u^2 + 2gh$

$v^2 = 20^2 + 2 \times (-10) \times (-10)$

$v^2 = 400 + 200 = 600$

$v = \sqrt{600} = 10\sqrt{6}\ ms^{-1}$

(iii) From the equation $h = ut + \frac{1}{2}gt^2$

$h = -10$ m, $g = -10\ ms^{-2}, u = 20\ ms^{-1}$ then

$-10 = 20t + \frac{1}{2}(-10)t^2$

$\therefore\ 5t^2 - 20t - 10 = 0$

Simplifying:

$t^2 - 4t - 2 = 0$

Solving for t:

$$t = \frac{4 \pm \sqrt{16 + 4 \times 2}}{2} = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2}$$

$\therefore\ t = \left(2 + \sqrt{6}\right)$ s

Attempt revision exercise 1 on pages 37 – 40 Longhorn Book Three

NON-LINEAR MOTION

Projectile Motion

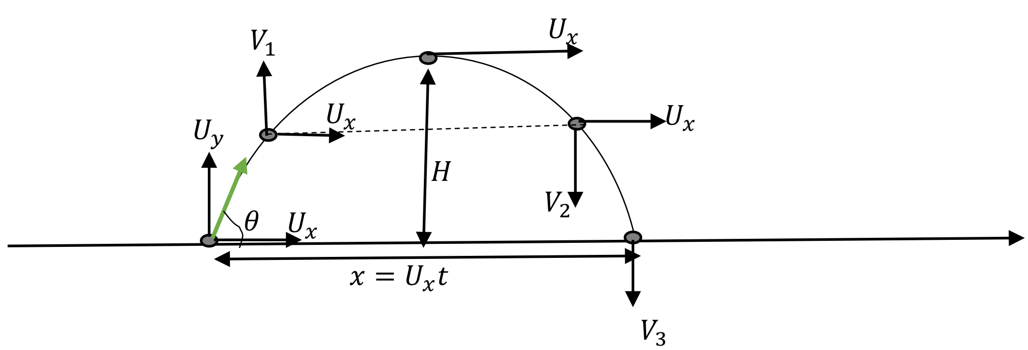
A **projectile** is a body given an initial velocity and allowed to move freely under the influence of gravitational force only.

Applications of projectile motion:

Projectile motion is applied in:

1. in ball games like football, netball, volleyball, basketball, etc.
2. in launching of missiles, cannons, etc.
3. in dropping of cargo from planes.

If a particle is projected at an angle, θ to the horizontal, its path will be a parabolic curve.



The particle's velocity at any instant will consist of two parts – the horizontal and vertical components.

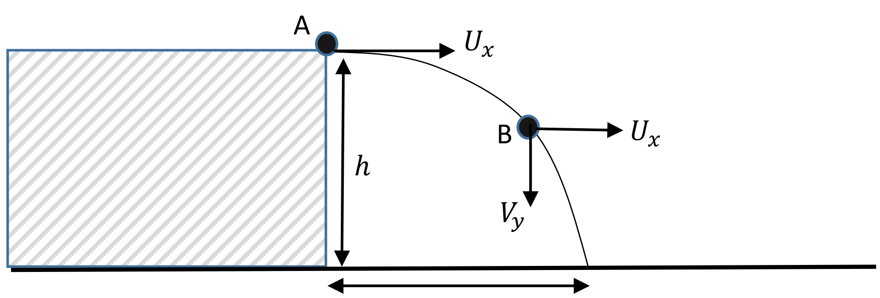
The projectile will experience both horizontal and vertical motion at the same time.

However, the horizontal motion is independent of the vertical motion.

Horizontal motion
The horizontal velocity, U_x remains constant throughout the motion. This is because there is no force of gravity acting in the horizontal direction.
The horizontal distance, x moved after time t is given by the equation $x = U_x t$.
The maximum horizontal distance moved by the projectile during the time of flight, T is known as the **range**.

Vertical motion.
The vertical motion of the projectile is influenced by the force of gravity which acts against it during its upward motion causing it to decelerate. At maximum height, H, the velocity of the projectile becomes zero. The projectile then changes direction and starts accelerating downwards.
The vertical distance, y moved after time t is given by the equation $y = \frac{1}{2}gt^2$.

Horizontal projection:
If an object is thrown horizontally, say from the top of a platform as shown below, with an initial velocity of U_x;



The object describes both horizontal and vertical motions that are independent of each other at the same time.

Horizontal motion
The horizontal motion is independent of the gravitational force, therefore the horizontal velocity, U_x remains constant throughout the motion.

horizontal distance = velocity × time of flight
$\mathbf{x = U_x t}$

Vertical motion.
When the object is at A, its initial vertical velocity is zero. However, the object accelerates uniformly under the influence of the gravitational force. Therefore, the vertical distance, h it covers in the time of flight is given by the equation $\mathbf{h = \frac{1}{2}gt^2}$.

Examples
1. A girl throws a ball horizontally from a window of a room on the 8th floor of a certain building. If it takes the ball 4 seconds to hit the ground below, find
   (i) the vertical height above the ground of the point of projection.

(ii) the velocity with which the ball was projected given that it landed 50 m away from the building.
(acceleration due to gravity, $g = 10\ ms^{-2}$)

Solution:

(i) Equation of vertical motion:

$$h = \frac{1}{2}gt^2.$$

Substituting in the equation;

$h = \frac{1}{2} \times 10 \times 4^2 = 80$ m.

(ii) Equation of horizontal motion:

$x = U_x t$

Substituting in equation:

$50 = U \times 4$

horizontal velocity of projection, $U = 12.5\ ms^{-1}$.

2. A UN plane travelling with a horizontal speed of $180\ ms^{-1}$ drops a parcel of supplies from a height of 2000 m above the ground. If acceleration due to gravity is $10\ ms^{-2}$, find,
   (i) the time taken by the parcel to reach the ground.
   (ii) the horizontal distance moved by the parcel from the time it was dropped from the plane.

Solution:

(i) From $h = \frac{1}{2}gt^2$,

$2000 = \frac{1}{2} \times 10 \times t^2$

$t^2 = 400$

$t = 20$ s

(ii) From $x = U_x t$

$$x = 180 \times 20 = 3600 \text{ m}.$$

3. A body is projected horizontally off the cliff at a velocity of $15ms^{-1}$. The height of the cliff is 20m.
   (a) Find the time it takes to reach the ground.
   (b) Find the distance from the cliff to where it falls.
   Leave 10 lines

Test yourself

1. A helicopter delivering relief food is travelling at $200ms^{-1}$ at a height of 500m.
   (i) Calculate the time food takes to reach the ground.
   (ii) Calculate the distance the food travels before it reaches the ground.

2. A ball goes down a ramp and is projected horizontally off the end of the table. If it falls a vertical height of 0.45m and hits the ground 2.1m away at point G.
   (a) How long does it take to fall 0.45m?
   (b) What is its horizontal velocity as it leaves the ramp?

3. A stone is thrown horizontally at $15 ms^{-1}$ from the top of a building of a height 125 m to a target on the ground. Calculate.
   (i) The time taken for the stone to hit the target.
   (ii) How far is the target from the foot the of the building.

4. A train travelling at a constant acceleration of 2 ms^{-2} passes a point A with a speed of 5 ms^{-1} and passes another point B 80 m ahead of A. Find the velocity of the body at B.

5. Two vehicles P and Q, originally at the same place, accelerate uniformly from rest. P attains a maximum velocity of 25 ms^{-1} in 10 s while B attains a maximum velocity of 40 m s^{-1} in the same time. Both vehicles maintain the same velocities respectively for 8 s. They then undergo uniform retardation such that P comes to rest in 4 s while Q comes to rest in 6 s.
   Find:
   (i) the velocity of each vehicle 18 s after start.
   (ii) the distance between the two vehicles when Q comes to rest.

6. A particle, which is retarding uniformly, passes a point A with a velocity of 40 m s^{-1} and after 4 s seconds it passes another point B 100 m ahead.
   Find
   (i) the acceleration of the particle
   (ii) how far the particle is from B when it comes to rest.

7. The table below shows the distance, x, in metres covered after time, t, in seconds for a moving particle.

t(s)	0	2	4	6	8	10
x(m)	4	14	24	34	44	54

   Plot a graph of distance against time and find the speed of the particle.

8. The table below shows the velocity v ms^{-1} attained after time t seconds for a particle.

t(s)	0	2	4	6	8	10	12	14	16	18
v(ms^{-1})	5	13	21	29	39	39	39	27	15	3

   Draw the velocity-time graph for the motion and describe the motion of the particle during its motion.
   Find:
   (i) The distance covered throughout the journey
   (ii) the acceleration of the particle
   (iii) the retardation
   (iv) the distance moved while accelerating
   (v) the time that will have elapsed when it stops

6. A particle is projected vertically upwards with a velocity of 30 m s^{-1}. Find:
   (i) the time taken for the particle to attain the greatest height.
   (ii) the displacement of the particle 5 s after projection.

Circular Motion

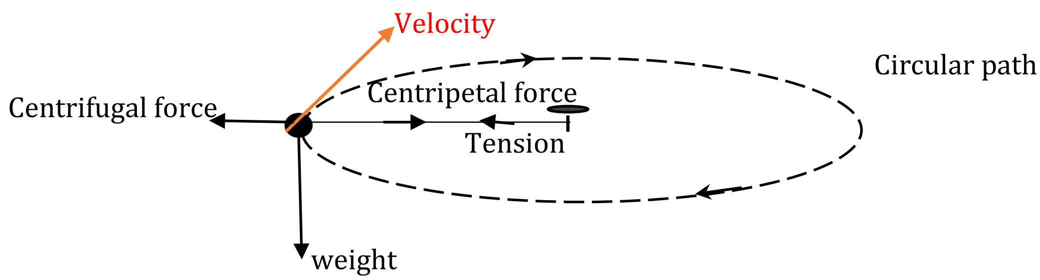
Examples of bodies moving in circular motion include;

1. A stone tied to one end of a string and the other end is rotated about a fixed point will describe a circular path.
2. A vehicle negotiating a circular path on a level horizontal road.
3. A cyclist negotiating a bend on a road, etc.
   Usually, the passengers in a vehicle that is negotiating a bend on the road tend to be drawn towards the centre of the bend.

Centripetal force.

This is the force that acts on a body keeping it rotating in a circular path and it acts towards the centre of the rotation.

Horizontal circular motion:



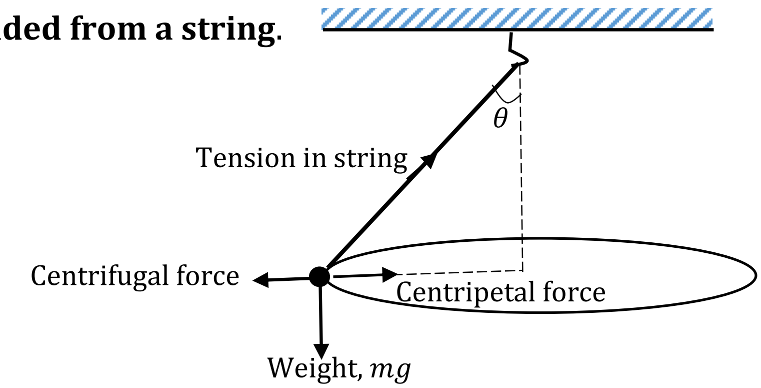
In uniform circular motion the speed of the particle is constant but the velocity is constantly changing because of change in direction. The acceleration is always perpendicular to the velocity. Hence, the acceleration is towards the centre of the circle.
The force accelerating the body towards the centre of the circle is called the **centripetal force.**
The force created by the moving body to oppose the centripetal force is called the **centrifugal force.** Centripetal force increases with:-

- Mass of the body
- Speed of the body

If the string to which the body is attached suddenly breaks, the body will fly off along a straight path that is along the tangent to the circular path at that point.
The forces acting on a body that is rotating in a horizontal circular path are:

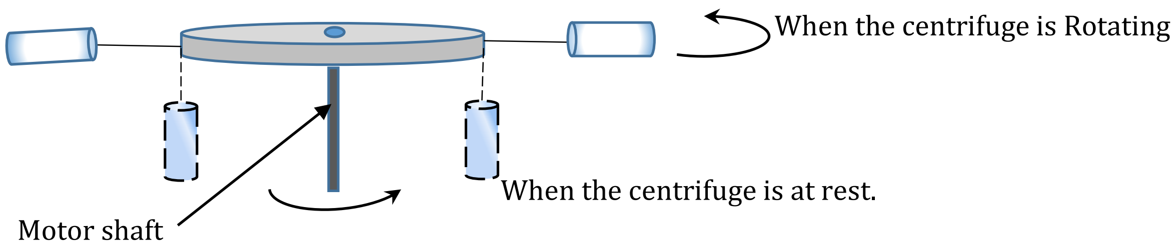
1. Centripetal force – this acts towards the centre of the rotation,
2. Tension in the string – this acts in the opposite direction of the centripetal force.
3. Weight of the body- this acts downwards.
4. Centrifugal force- this acts as a reaction force to the centripetal force.

Circular motion of a body suspended from a string.



When a body suspended on a string is set to rotate in a horizontal circular path. The angle, θ made by the string to the vertical gradually increases from 0^o to a certain value. The angle, θ increases with the speed of rotation.

The centrifuge:
A centrifuge is a device that that separates liquids of different densities or solids suspended in liquids. The mixture is poured into a tube in the centrifuge which is then rotated at a high speed in a horizontal circle either mechanically or with the help of a motor.
The tube is initially in the vertical position and takes up the horizontal position when the centrifuge starts working as shown in the diagram below.



The matter of low density moves inwards towards the centre of rotation. On stopping the rotation, the tube returns to the vertical position with less dense matter at the top.

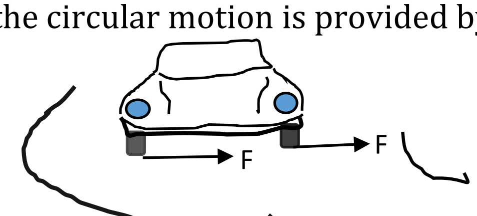
Working principle of the centrifuge:
The centrifuge works on the principle of force developed due to the pressure difference in the tube. The liquid pressure at the bottom of the tube is greater than the pressure at the top and a pressure gradient exists along the length of the tube.
The force due to the pressure difference supplies the required centripetal force in each part of the tube. The force at the bottom of the tube is greater and the matter of low density, having a smaller mass, moves inwards to have a smaller radius and liquid of high density (high mass) moves away from the axis of rotation towards the bottom. The liquids of different densities are thus separated.

Applications of the centrifuge:
1. Cream separators: When the milk is churned rapidly, the lighter cream moves towards the top of the tube from where it is removed.

2. Blood Testing: When blood is rotated at high speed, red blood cells and the blood fluid are separated. Viruses and germs in the blood fluid cab be separated in a similar manner.

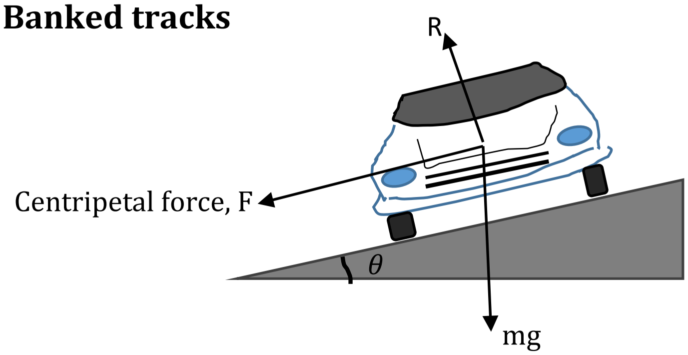
3. Very high speed centrifuges, called ultra- centrifuges are used in medical research e.g. in the study of viruses like HIV.

Applications of circular motion in a horizontal circle.
1. When a car is going round a circular path on a horizontal road, the centripetal force required for the circular motion is provided by the frictional force, F, between the tyres and the road.



This force prevents the car from sliding even when it is moving very fast.

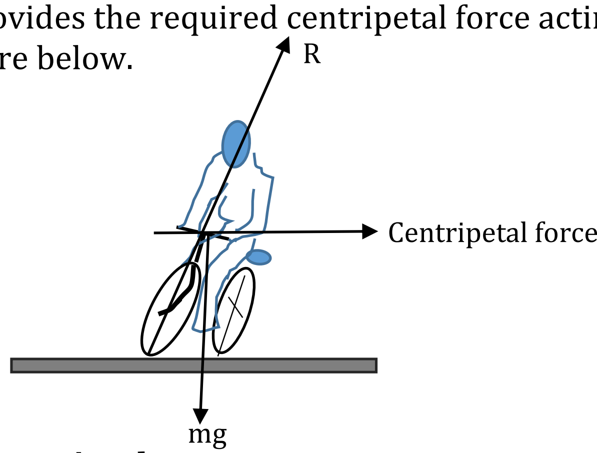
2. **Banked tracks**



In order that a car does not fully depend on the frictional force between the tyres and the road, circular paths are given a small banking edge, θ i.e. the outer edge of the road is raised a little above the inner side so that the track is sloping towards the centre of the curve. The figure above shows a part of the contact force R (the normal reaction force) acting towards the centre of the circle providing the required centripetal force.

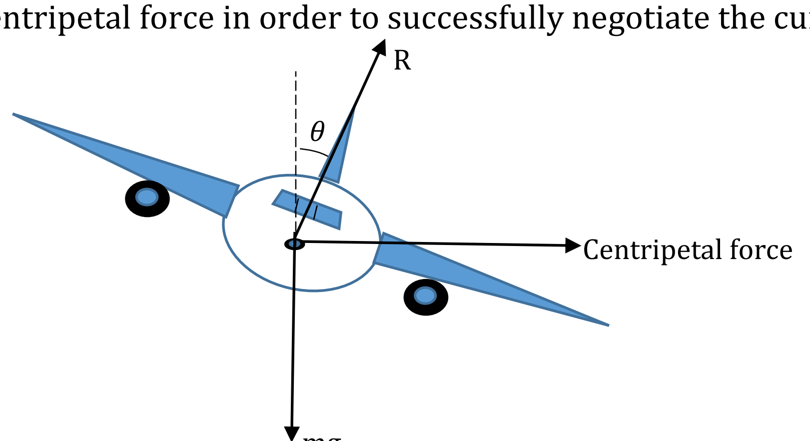
A cyclist going round a curve.

A cyclist going round a curve leans inwards to provide the necessary centripetal force so as to be able to go along the curved track. Just like a car on a banked track, a part contact force or the reaction force provides the required centripetal force acting towards the centre of the track, as shown in the figure below.

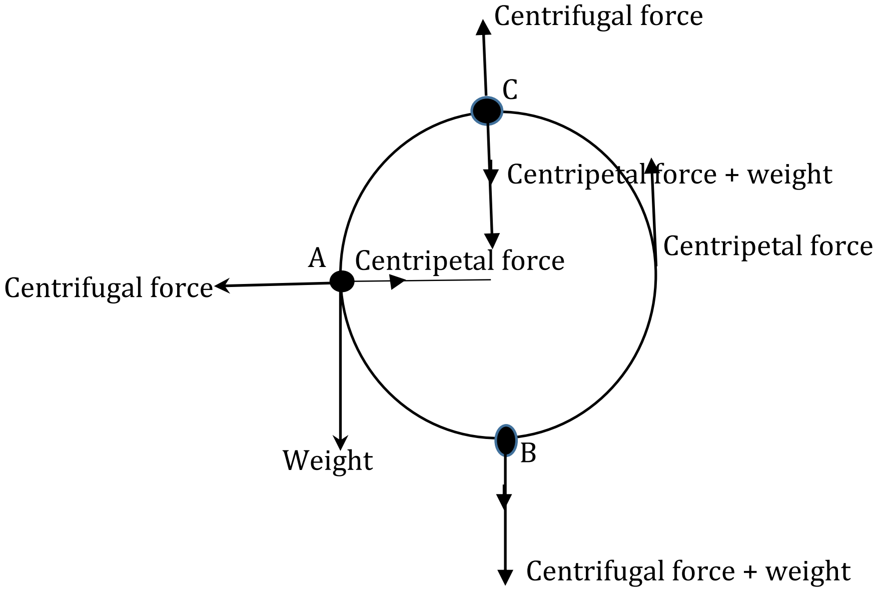


An aircraft taking a circular turn

When an aircraft takes a turn in a horizontal plane, it must maintain some speed and make a correct banking angle in mid-air as shown in the diagram below, so that its weight provides the necessary centripetal force in order to successfully negotiate the curved path.



Vertical Circular motion.



The diagram above shows the direction of the different forces acting on a body moving in a vertical circular motion at different positions A, B and C respectively.

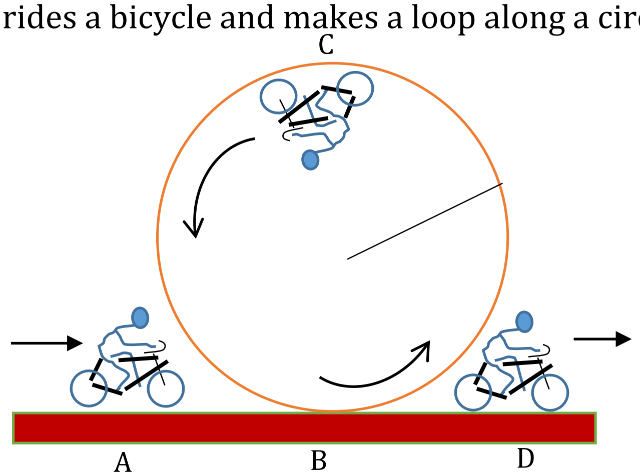
The tension, T in the string at any of the positions A, B and C of the body is equal to the centrifugal force at that point.

i.e. At position A, Tension, T_A = centripetal force alone.
At position B, Tension, T_B = centripetal force − weight.
At position C, Tension, T_C = centripetal force + weight.

Tension is least at B and greatest at C. Therefore, the string is most likely to break at C than at any other point.

Applications of motion in a vertical circle.

In a circus, an acrobat rides a bicycle and makes a loop along a circular track in a vertical plane, as shown below:



The cyclist will stay on the vertical track as long as he maintains a speed that gives a centripetal force greater than or equal to mg, the weight of the bicycle and cyclist.

Similarly, a bucket of water can be swung round a vertical circle without spilling the water. The water in the bucket will stay in the track as long as the centripetal force is greater than or equal to the total weight of water and the bucket.

Drying machine

Wet clothes are rotated to in a cylindrical drum containing a lot of perforations.

Initially, the wet clothes move in a circular motion along with the drum. As the speed of the drum increases, the adhesive force of the water in the clothes decreases and the water beaks off from the clothes and flies off through the perforations, hence, the clothes become dry.

CENTRIFUGES.

A centrifuge is a device that separates liquids of different densities or solids suspended in liquids

Summary of applications of uniform circular motion.

1. A car negotiating a circular path on a level horizontal road
2. Banked tracks
3. A cyclist going round a curve
4. An aircraft taking a circular turn.
5. Drying machines for clothes
6. Centrifuges

Example one.

2012 p2 no. 1 – Leave 15 lines.

Attempt Revision Exercise 2 on page 52 Longhorn Book three.

SUB-TOPIC: SCALAR AND VECTOR QUANTITIES

SPECIFIC OBJECTIVES:

- Define vector and scalar quantities,
- State examples of vector and scalar quantities.
- Find the resultant of vectors.
- Find the perpendicular components of a vector.

SCALAR QUANTITIES

A Scalar quantity is one that is fully describes by its size (or magnitude) only.

Examples of Scalar Quantities include:

Time, distance, mass, density, area, volume, speed, energy, work, power, etc.

The value attached to scalar quantities represents their magnitude or size.

Addition of scalar quantities:

The scalar quantities to be added must be identical

e.g. (i) $5\text{ kg} + 6\text{ kg} = (5 + 6)\text{ kg} = 11\text{ kg}$

(ii) $4.6\text{ m} + 9.4\text{ m} = (4.6 + 9.4)\text{m} = 14.0\text{ m}$

VECTOR QUANTITIES

A vector quantity is one that is fully described by both magnitude and direction.

Examples of vector quantities include:

Velocity, displacement, acceleration, force, moment of a force, momentum, etc.

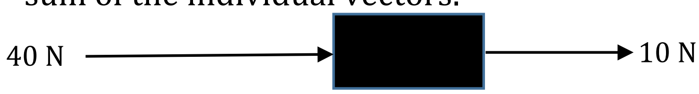
A vector quantity is represented as a line with an arrow.
When drawn to scale, the length of the line represents the magnitude, while the arrow represents the direction of the vector quantity.

Addition or subtraction of vector quantities is carried out vectorially.

Vector Addition
The sum of a number of vectors is known as the **resultant** of the vectors involved.

The resultant vector is a single vector having the same effect as the vectors from which it is derived.

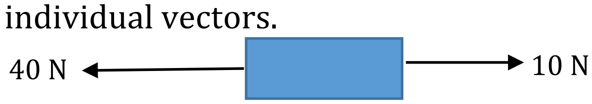
(a) **Resultant of two vectors acting in the same direction**:
When two or more vectors are acting in the same direction, their resultant is equal to the sum of the individual vectors.



The resultant force of the two forces acting on the object shown above is obtained as follows:

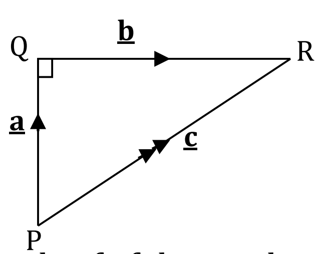
$$F = (40 + 10)\ N = 50\ N$$

(b) **Resultant of two vectors acting in opposite directions**:
When two vectors are acting in opposite directions, their resultant is equal to the difference of the individual vectors.



The force point to the right is taken as being positive and that point to the left is taken as being negative.
The resultant force of the two forces acting on the object shown above is obtained as follows:
$F = (-40 + 10)\ N = -30\ N$
The resultant force is equal to 30 N in the direction of the 40 N force.

(c) **The resultant of two vectors acting at right angles to each other.**
Imagine two vectors **a** and **b** which are perpendicular to each other. Their resultant, **c** is given by $\mathbf{c} = \mathbf{a} + \mathbf{b}$

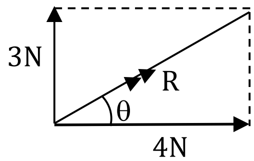


The magnitude of of the resultant force **c** is PR, and is given by Pythagoras theorem
$$\mathbf{PR^2 = QP^2 + QR^2}$$
$$\therefore\ \mathbf{PR = \sqrt{QP^2 + QR^2}}$$

Examples

Two forces of 3 N and 4 N, at right angles, act at a point. Find their resultant.

Solution



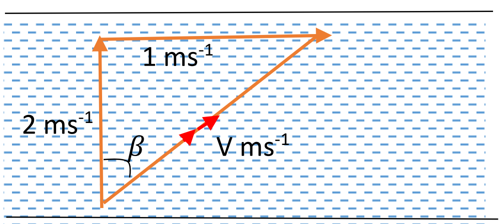
The resultant, R $= \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ N at an angle θ as shown

$\tan\theta = \frac{3}{4}$

$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^o$

2. A man who can swim at 2 m s^{-1} in still water swims directly across a river that flows at 1m s^{-1}. What is the resultant velocity of the man?

Solution

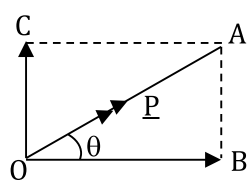


The resultant velocity, v $= \sqrt{2^2 + 1^2} = \sqrt{5} = 2.24\ ms^{-1}$ at an angle β, where

$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^o$

Resolution of Vectors

A vector may be expressed in terms of two vectors that are perpendicular to each other. When this is done, the vector is said to have been **resolved** into two vectors. The two vectors so obtained are known as **components** of the vector. For example:

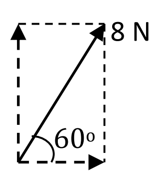


Imagine a vector P, along OA is resolved into components acting along OB and OC. The component acting along OB has magnitude **P cosθ** and that along OC **P sinθ**.

Example

A force of 8N acts at an angle of 60° to the horizontal as shown in the diagram below. Find the magnitude of its horizontal and vertical components.

Solution



Magnitude of horizontal component = 8 cos 60^o

= 4 N

Magnitude of the vertical component = 8 sin 60^o

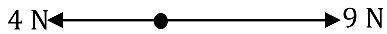
= 6.93 N

Attempt Revision Exercise 3 on pages 57 – 61

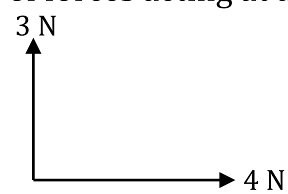
Test Yourself

1. Determine the resultant of each the following combinations of forces acting at a point

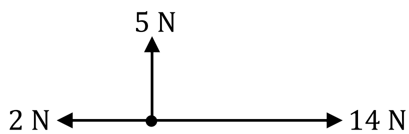
(a)



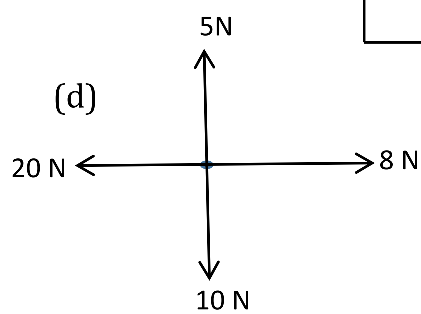
(b)



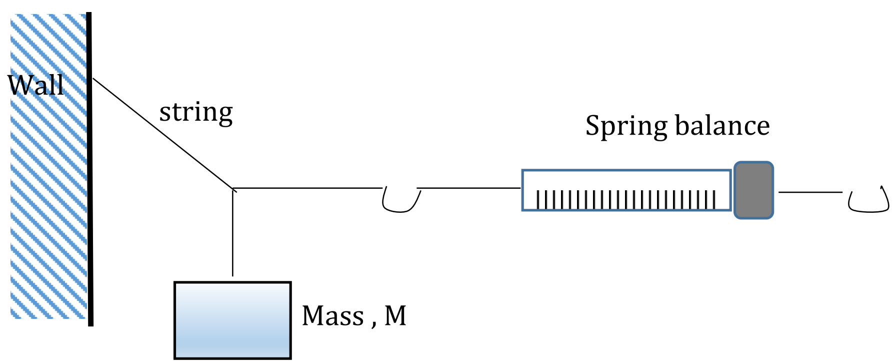
(c)



(d)



2. A mass of 20 kg is held in equilibrium by a string fixed on the wall and a horizontal spring balance as shown in the figure below.



If the spring balance reads 20 N, determine

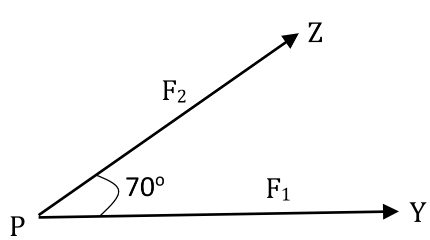
(i) the tension in the string.

(ii) the mass, M of the body.

3. Two forces, F_1 and F_2, are acting at the point P along the directions PY and PZ as shown in the figure below. If F_1 = 10 N and F_2 = 5 N, find by scale drawing or otherwise,

(i) the magnitude of the resultant force.

(ii) the angle between the resultant and F_1.



SUBTOPIC: LINEAR MOMENTUM
SPECIFIC OBJECTIVES

- Define linear momentum and state its unit.
- State the law of conservation of linear momentum.
- Solve numerical problems using the law of conservation of linear momentum.
- Describe situations where linear momentum is applied.

MOMENTUM AND COLLISIONS

Any moving body is said to possess **momentum**.
Definition:
Linear momentum of a body is the product of mass of the body and its velocity.
Therefore, a body of mass **m** moving with a velocity **v** has momentum equal to **mv**.
i.e. momentum, $p = \text{mass} \times \text{velocity}$

$$\therefore\ p = mv$$

The SI unit of momentum is **kilogram metre per second** ($\boldsymbol{kgms^{-1}}$).

Linear momentum is a vector quantity. The direction of momentum is the same as that of the velocity.
When a force is applied to a body, it can change its momentum.

Factors affecting momentum of a body.

1. Mass of the body
2. Velocity of the body.

The momentum of a body increases with both its mass and velocity.

Examples.

1. A heavy hammer can drive a nail deeper into a piece of wood than a lighter hammer.
2. Once in motion, a heavy truck is harder to stop than a smaller car. The reverse is also true.

Examples

1. Find the momentum of a car of mass 600 kg moving with a constant velocity of 30 ms^{-1}.
   Solution
   Momentum = mass × velocity
   $= 600 \times 30 = 18000\ kgms^{-1}$ in the direction of the velocity.

2. A truck of mass 1200 kg initially moving with a velocity of 15 ms^{-1} accelerated uniformly at a rate of 1.5 ms^{-2} for 10 s .Find
   (i) its final velocity after the 10 s.
   (ii) its momentum after the 10 s.
   (iii) the difference in momentum.
   Solution
   (i) $v = u + at$
   $v = 15 + 1.5 \times 10 = 15 + 15$
   $v = 30\ ms^{-1}$.

(ii) final momentum $= \text{mass} \times \text{final velocity}$
$= 1200 \times 30$
$= 36000\ \text{kgms}^{-1}$.

(iii) change in momentum $= \text{final momentum} - \text{initial momentum}$
$= mv - mu$
$= m(v - u)$
$= 1200(30 - 15) = 1200 \times 15$
$= 18000\ \text{kgms}^{-1}$.

3. A body of mass 4kg moves to the left with a velocity of 7 ms^{-1}. Another body B of mass 7kg moves to the right with a velocity of 6 ms^{-1}. Calculate
(a) the momentum of A (-28 kgms^{-1})
(b) the momentum of B (42 kgms^{-1})
(c) the total momentum. (14 ms^{-1}) to the right.

IMPLUSE

When a force, F acts on an object for a very short time, t, it produces an impact, usually referred to as an impulse on the object.

Impulse of a force is the product of mass and time of action of the force on the object.
Therefore, Impulse $= \text{force} \times \text{time}$
$\text{impulse} = Ft$.
SI unit of impulse is a newton second (Ns).

When an impulsive force acts on an object, it produces a change in momentum of that object. The impulsive force changes the velocity of the object but its mass remains constant.

$$\text{impulse} = \text{change in momentum}$$
$$Ft = mv - mu$$

Example.

1. (a) A footballer kicks a ball pf mass 0.25 kg and initially at rest with a force of 200 N that acts for 0.5 s when taking a penalty kick. Find
(i) the impulsive of the force on the ball.
(ii) the takeoff velocity of the ball.
Solution:
(i) $\text{impulse} = \text{force} \times \text{time}$
$\text{impulse} = 200 \times 0.5$
$\text{impulse} = 100\ \text{Ns}$

(ii) $\text{impulse} = \text{change in momentum}$
$Ft = mv - mu$
$100 = 0.25 \times v - 0.25 \times 0$
$v = \frac{100}{0,25}$
$v = 400\ \text{ms}^{-1}$.

2. A goalkeeper is to catch a ball of mass 0.25 kg travelling at 250 ms^{-1}.
Find the impulsive force exerted on the goal keeper's hands
(i) if the impact lasts for 0.2 s.
(ii) if the impact lasts for 1 s when the goal keeper draws his hands towards his body as he catches the ball.

NEGATIVE EFFECTS OF IMPULSIVE FORCES:

Impulsive forces are forces that act for a very short time on an object.

1. Impulsive forces may lead to bodily harm, for example, when a freely falling body is suddenly stopped when it hits the ground.
2. Impulsive forces tend to change the shapes of colliding bodies when the collision lasts for a very short time.

Limiting the negative effects of impulsive forces.

This is achieved by prolonging the time of action of the force on the object.
For example:

1. A goalkeeper draws his hands towards his body when catching a fast moving ball. The takes the 'sting' or hurting effect as the ball is caught.
2. Shock absorbers reduce the force exerted on the object or vehicle as it travels over pot holes.
3. The nets at the back of a goal post are made loose to increase the time of action as the ball hits the net. This prevents the nets from tearing.
4. Goalkeepers/wicketkeepers wear soft gloves that absorb the shock and reduce the force on the hands by increasing the time of action of the force.
5. Materials that easily break like eggs, glassware etc. are packed in soft, shock-absorbing boxes. This reduces the possibility of them cracking on sudden stop or start of motion. Seat belts in cars use the same principle.
6. High jumpers usually bend their knees on landing. This increases the time of impact, hence, reducing the chances of injuries. In addition, the jumpers land in sand or soft cushions that increase the time of action and absorb the shock of impact.
7. In golf and lawn tennis, players follow the ball as it is hit. This increases the time of contact hence increases the velocity of the ball. At the same time, it reduces the reaction force the player feels on hitting the ball.
8. Cars are fitted with air bags that reduce the shock of impact in the course of an accident. Cars are also fitted with glass that stretches on impact and a crumple zone. This increases the time of impact, hence, reducing injuries.

Applications of impulsive forces:

Impulsive forces are usually applied in:

1. Rockets and jet engines of planes to produce very large forces of propulsion.
2. Guns to provide a high velocity to the bullet when the gun is fired.

Conservation of linear momentum and Collisions

When bodies collide, each body exerts a force on the other. The forces act for the same time interval but in opposite directions. So, the momentum of each body changes.

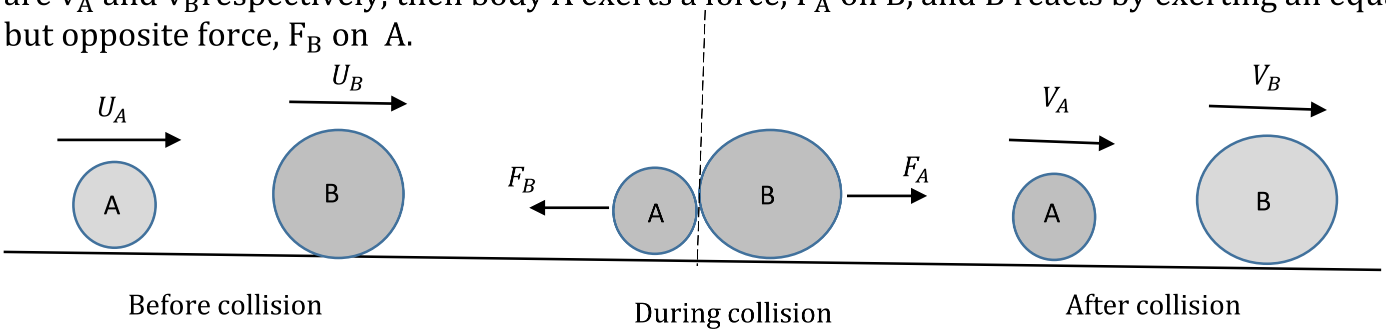
However, as long as no other force participates in the impact, the changes in momentum are such that the total momentum of the bodies remains the same.

Law of Conservation of Momentum states that:
If no external force acts on a system of colliding bodies, the total momentum of the bodies remains constant.

This law can be restated as:
For any system of colliding bodies, their total momentum before collision is equal to their total momentum after collusion, provided no external force acts on the system.

This may be summarized as follows:
Total momentum before collision = total momentum after collision

Consider two Bodies, A and B each of mass m_A and m_B and moving with velocities, u_A and u_B , respectively before collision. If their collision lasts, t seconds and their velocities after the collision are v_A and $v_B$respectively, then body A exerts a force, F_A on B, and B reacts by exerting an equal but opposite force, F_B on A.



During collision;
Force exerted on A by B = Force exerted on B by A
$\therefore \quad F_B = -F_A$
But, the forces act for the same time, t on both the bodies, therefore, their impulse is given as,
$F_B t = -F_A t$
Hence,
Change in momentum of body, B = Change in momentum of body, A
$\mathbf{m_B V_B - m_B U_B = -(m_A V_A - m_A U_A)}$

Reorganizing the above equation gives
$\mathbf{m_A U_A + m_B U_B = m_A V_A + m_B V_B}$
i.e.
Sum of momentum before collision = sum of momentum after collision.
Thus confirming the law of conservation of momentum.

Collisions
There are two types of collisions, namely **elastic** and **inelastic** collisions.

Elastic collisions.
A collision is said to be an **elastic collision** if the colliding bodies do no stick together after the collision**.**

In an elastic collision, both **momentum** and **kinetic energy** of the colliding bodies are conserved.

The equations of an elastic collision are therefore as follows:

1. momentum before collision = momentum after collision

$$\therefore\ \mathbf{m_A U_A + m_B U_B = m_A V_A + m_B V_B}$$

2. Kinetic energy before collision = kinetic energy after collision.

$$\therefore\ \therefore\ \mathbf{\frac{1}{2}m_A {U_A}^2 + \frac{1}{2}m_B {U_B}^2 = \frac{1}{2}m_A {V_A}^2 + \frac{1}{2}m_B {V_B}^2}$$

Inelastic collisions

When colliding bodies stick together and move as one body after the collision, they are said to have undergone **inelastic collision**. Their total **momentum is conserved** but their **kinetic energy is not**.

The equations of inelastic collision are as follows:

1. momentum before collision = momentum after collision

$$\therefore\ \mathbf{m_A U_A + m_B U_B = (m_A + m_B)V}$$

2. Kinetic energy before collision = kinetic energy after collision.

$$\therefore\ \therefore\ \mathbf{\frac{1}{2}m_A {U_A}^2 + \frac{1}{2}m_B {U_B}^2 = \frac{1}{2}(m_A + m_B)V^2}\ .$$

Causes of loss of kinetic energy during an elastic collision:

The following occurrences during an inelastic collision are the major causes of loss of kinetic energy during an inelastic collision.

1. **Heat**: some of the initial kinetic energy of the colliding bodies is converted to internal heat energy of the bodies leading to an increase in their temperature.
2. **Sound**: another fraction of the initial kinetic energy is converted to sound energy as the bodies collide.
3. **Deformation or change of shape** of the colliding bodies. Some energy is also lost in deforming or changing the shape of the bodies as they collide.

The comparisons between elastic and inelastic collision can be summarized in the table below:

Factor	Elastic collision	Inelastic collision
Momentum	is conserved.	is conserved.
Kinetic energy	is conserved.	is not conserved.
Velocity	Bodies have different velocities after collision.	Bodies have the same velocity after collision.
Behaviour of bodies after collision	Bodies separate after collision.	Bodies stick together after collision.

Examples

1. A particle P of mass 1 kg moving with a velocity of 2 m s^{-1} is knocked directly from behind by another particle Q of mass 2 kg moving at 4 ms^{-1}. If the velocity of P increases to 4.5 ms^{-1}, find the new velocity of Q.

Solution

$m_1 = 1$ kg, $u_1 = 2$ m s^{-1}, $v_1 = 4.5\ ms^{-1}$

$m_2 = 2$ kg, $u_2 = 4\ ms^{-1}$, $v_2 = ?$

Total momentum after collision = total momentum before collision

$\therefore \quad m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$

$(1 \times 4.5) + 2 \times v_2 = (1 \times 2) + (2 \times 4)$

$\therefore \ 4.5 + 2v_2 = 2 + 8$

$\therefore v_2 = 2.75\ ms^{-1}$

2. A ball X of mass 1 kg moving with a velocity of 3 m s^{-1} collides directly with another ball Y of mass 2 kg moving at 2 ms^{-1}in the opposite direction. If Y reverses at 1 ms^{-1}, find the new velocity and direction of motion of X after collision.

Solution

Let the initial direction of X be positive and its mass m_1.

$m_1 = 1$ kg, $u_1 = 3$ m s^{-1}, $v_1 = ?$

$m_2 = 2$ kg, $u_2 = -2$ m s^{-1}, $v_2 = 1$ m s^{-1}

Total momentum after collision = total momentum before collision

$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$

$(1 \times v_1) + (2 \times 1) = (1 \times 3) + (2 \times -2)$

$\therefore v_1 = 3 - 4 - 2 = -3\ ms^{-1}$

The negative sign means that X moves in the opposite direction of its motion before collision occurred at a velocity of 3 m s^{-1}

3. An arrow of mass 100 g moving at a velocity of 16 m s^{-1} horizontally enters a block of wood of mass 540 g lying at rest on a smooth surface. Find the common velocity after impact.

Solution

$m_1 = 100$ g, $u_1 = 16$ m s^{-1}, $v_1 = v_2 = v$ i.e. the common velocity of the two bodies, since they move together after collision.

$m_2 = 540$ g, $u_2 = 0$

Total momentum after collision = total momentum before collision

$(m_1 + m_2)v = m_1u_1 + m_2u_2$

$\therefore \ (100 + 540)v = 100 \times 16$

$\therefore \quad v = 2.5\ ms^{-1}$

Test Yourself

1. A trolley P of mass 150g moving with a velocity of $20ms^{-1}$ collides with another stationary trolley X of mass 100g.. P and X move together after collision. Calculate,
   (i) momentum of P before collision.
   (ii) the velocity of P and X with which they moved after collision.
2. A mass of 3kg moving with a velocity of 4 ms^{-1} collides with another mass of 2kg which is stationary. After collision, the two masses stick together. calculate the common velocity of the masses. ($2.4ms^{-1}$)
3. A 5kg mass moving with a velocity of $10ms^{-1}$collides with a 10kg mass of a velocity of $7ms^{-1}$ along the same line. If the two masses join together on impact, find their common velocity if
   (a) Were moving in the same direction. ($8.0\ ms^{-1}$)
   (b) Were moving in the opposite direction. ($-1.33ms^{-1}$)

Attempt Revision Exercise 4 on pages 69 - 70 in Longhorn Book three

Case of Gun and Bullet

Before the bullet is fired, the total momentum of gun and bullet is zero. Therefore, even when the bullet is fired, the total momentum must remain zero, since the total momentum is conserved. This is why when the bullet moves forward, the gun must move backwards (recoil) with an equal but negative momentum.

Let

M_g = mass of the gun.

m_b = mass of the bullet.

U_g = initial velocity of the gun.

V_g = final velocity of the gun = the recoil velocity.

u_b = initial velocity of the bullet

v_b = final velocity of the bullet

Before pulling the trigger.

$U_g = 0\text{ms}^{-1}$

$u_b = 0\text{ms}^{-1}$

Applying the principle of conservation of momentum

Total momentum after collision = total momentum before collision

$$M_gU_g + m_bu_b = -M_gV_g + m_bv_b$$
$$M_g \times 0 + m_b \times 0 = -M_gV_g + m_bv_b$$

$$0 = -M_gV_g + m_bv_b$$
$$M_gV_g = m_bv_b$$

Example

1. A gun of mass 4 kg fires a bullet of mass 50 g at a velocity of 200 m s^{-1}. Find the recoil velocity of the gun.

   Solution

   The recoil momentum of the gun must equal to the forward momentum of the bullet. Let the recoil velocity be V.

   Then,

   Recoil momentum of gun = momentum of the bullet.

   $$M_gV_g = m_bv_b$$
   $$4000\text{V} = 50 \times 200$$

   ∴ V = 2.5 ms^{-1}, is the recoil velocity of the gun.

2. A bullet of mass 6 g is fired from a gun of mass 500 g. If the muzzle velocity of the bullet is 300 ms^{-1}, calculate the recoil velocity of the gun. (Ans = $\mathbf{v_g} = \mathbf{-3.6}$ ms^{-1}

   Since velocity is a vector quantity, the minus sign indicates that the bodies move to the left (i.e. in the original direction of body B) after collision. Therefore, the gun kicks backwards with a velocity of 3.6 ms^{-1})

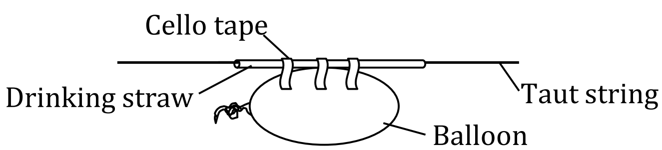
3. A gun of mass 5 kg fires a bullet of mass 50 g at a speed of 500 ms^{-1}. Calculate the recoil velocity of the gun. (5 ms^{-1})

Solution: Mass of the gun, mg = 5 kg, Mass of bullet, m_b = 50 g = $\frac{50}{1000}$ kg

Initial velocity, u_g, of the gun = 0
Initial velocity, u_b, of the bullet = 0
Final velocity, v_g, of gun =?
Final velocity, vb, of the bullet = 500 ms-1
We take the direction of the bullet to be positive and that of gun to be negative.
Then we can solve the problem by using any one of the following methods.

Applications of the principle of Conservation of momentum
The principle of conservation of momentum is applied in:
(i) Rocket propulsion
(ii) Jet engine
(iii) Turbines
(iv) Rowing a boat

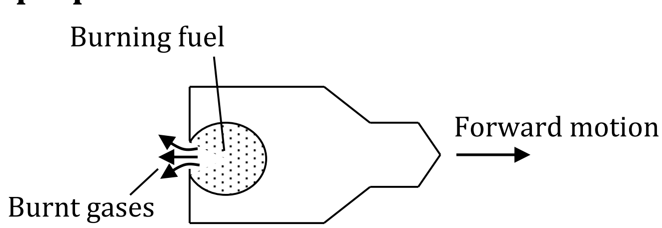
Experiment : To Demonstrate Rocket Propulsion using an inflated balloon.
A balloon is inflated and its opening is sealed by twisting it.
A drinking straw is taped lengthwise along one side of the balloon
A string is ran through the straw.
One end of the string is attached to any rigid stationary object.
The string is pulled taut and its other end is attached to another stationary object across the room (or you may have a colleague hold the other end of it)



Once the setup is complete, the twist is untied and the balloon is released.s
Observation
The balloon will be observed to move in the opposite direction of the escaping air.

The experiment is repeated but this time the string is inclined so that the twisted end of the balloon is lowermost before releasing it.
Explanation
When the balloon is opened the air molecules gain momentum in the direction of the exit at a certain rate. This results in creation of a force pushing the balloon in the opposite direction. So, it is propelled forward.

(a) Rocket propulsion



A space rocket carries tanks of liquid fuel and liquid oxygen and some chemicals which react to produce oxygen to enable the fuel to burn.
When the fuel burns inside the rocket engine, it creates a large force which propels a blast of hot gaseous products of the combustion out through the tail nozzle with very high velocity.
By Newton's third law of motion, the reaction to this force propels the rocket forward.
Note that although the mass of the gas emitted per second is very small, it has a very large momentum on account of its high velocity. An equal momentum is imparted on to the rocket in the opposite direction, so that in spite of its large mass, it also builds up a high velocity.

(b) Jet engine

A jet engine (used on air crafts) operates on the same principle as rocket propulsion. As the air craft does not leave the earth's atmosphere, oxygen supply is from the air.
The fuel burns to form large quantities of gaseous products. As the blast of hot gas molecules (burnt fuel and excess air or oxygen) is thrown out from the combustion chamber through the exhaust pipe with high momentum, the jet in turn, acquires an equal momentum but in the opposite direction enabling it to move forward.

Attempt Exercise 5.3 on pages 85-86

Topical Revision Exercise

Section A

1. When a car is suddenly brought to rest, a passenger jerks forward because of
A. inertia B. friction C. gravity D. momentum

2. A boxer while training noticed that a punch bag is difficult to set in motion and difficult to stop. What property accounts for this observation?
A. Size. B. Inertia. C. Friction. D. Weight of the bag.

3. Eggs packed in a soft, shock-absorbing box are placed in a car. When the car suddenly starts or stops moving, the eggs do not crack because
A. no force acts on them
B. the force acts on them for only a short time
C. the force is small and acts for a longer time
D. the force causes fast change of momentum.

4. A body of mass 20 kg moves with a uniform velocity of 4 m/s from rest. Find its momentum.
A. 5 kg ms^{-1} B. 80 kg ms^{-1} C. 160 kg ms^{-1} D. 320 kg ms^{-1}

5. An object of mass 2 kg moving at 5 ms-1, collides with another object of mass 3 kg which is at rest. Find the velocity of the two bodies if they stick together after collision
A. 1.0 ms^{-1} B. 2.0 ms^{-1} C. 2.5 ms^{-1} D. 5.0 ms^{-1}

6. A bullet of mass 0.1 kg is fired from a rifle of mass 5 kg. The rifle recoils at a velocity of 16 ms^{-1}. Calculate the velocity with which the bullet is fired
A. 66 ms^{-1} B. 110 ms^{-1} C. 210 ms^{-1} D. 800 ms^{-1}

7. A body of mass 20 kg moves with a uniform velocity of 4 ms^{-1} from rest. Find its momentum.
A. 5kgms^{-1} B.80 C.160 D. 320

8. When a person steps forward from rest, one foot pushes backwards on the ground. The ground will as a result push that foot

A. backwards with an equal force B. forwards with an equal force
C. backwards with a smaller force D. forwards with a smaller force

9. If the forces acting on a moving body cancel each other out (i.e. are in equilibrium) the body will
A. Move in straight line to the steady speed
B. Slow down to a steady slower speed
C. Speed up a steady faster speed
D. Be brought to a state of rest.

10. A body of mass 20 kg, moving with uniform acceleration, has an initial momentum of 200kg $ms^{-1}$and after 10s, the momentum is 300 kg m/s. What is the acceleration of the body?
A. 0.5 ms^{-2} B. 5 ms^{-2} C. 25 ms^{-2} D. 50 ms^{-2}

SECTION B

1. (a) State Newton's laws of motion.
(b) Define: (i) Inertia of a body (ii) Momentum.
(c) Explain why a passenger standing on the floor of a lorry jerks backwards when the lorry starts moving forwards.
(d) A 7-tonne initially moving at a velocity of 50m/s accelerates to 80m/s in 3 seconds. Calculate the force on the truck that caused the velocity change.

2. (a) (i) What is meant by linear momentum?
(ii) State the law of conservation of linear momentum.
(b) A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of 20 ms^{-1}, calculate.
(i) The speed with which the bullet hits the wood,
(ii) The kinetic energy lost.
(c) State the energy changes involved in (b) above.

3. (a) State the law of conservation of linear momentum.
(b) A water jet directed to a spot on the ground digs a hole in the ground after sometime. Explain.
(c) A moving ball P of mass 100g collides with a stationary ball Q of mass 200g. After collision, P moves backwards with a velocity of 2ms-1 while Q moves forwards with a velocity of 5ms-1. Calculate
(i) The initial velocity of P.
(ii) The force exerted by P on Q if the collision took 0.03s.
(d) Explain the principal of operation of a rocket engine

4. A sphere of mass 3 kg moving with velocity 4m/s collides head on with a stationary sphere of mass 2kg & imparts to it a velocity of 4.5 m/s. Calculate the velocity of the 3kg sphere after the collision.

5. A railway tracks of mass 4x104 kg moving at a velocity of 3m/s collides with another truck of mass 2 x104 kg which is at rest. The couplings join & the trucks move off together.
(a) State the type of collision.
(b) Calculate the common velocity after collision.

6. A car of mass 1500 kg moving with velocity of 25 ms^{-1} collides directly with another car of mass 1400 kg at rest so that the two stick and move together. Find their velocity.

7. A bullet of mass 30 g is fired into a stationary block of wood of mass 480 g lying on a smooth horizontal surface. If the bullet gets embedded in the block and the two move together at a speed of 15 ms^{-1}. Find:
   (i) the speed of the bullet before it hits the block.
   (ii) the kinetic energy lost.

8. A moving ball A of mass 200 g collides directly with a stationary ball B of mass 300 g so that A bounces with a velocity of 2 m s^{-1} while B moves forward with a velocity of 3 ms^{-1}. Calculate the initial velocity of A.

4. A particle X of mass 2 kg originally moving with a velocity of 3 ms^{-1} collides directly with another particle Y of mass 2 kg which is moving at a velocity of 2 ms^{-1} in the opposite direction so that the velocity of X becomes 1 ms^{-1} after the impact. Find the velocity of Y after the impact.

5. A bullet of mass 40 g is fired with a velocity of 200 ms^{-1} from a gun of mass 5 kg. What is the recoil velocity of the gun?

SUB-TOPIC: NEWTON'S LAWS OF MOTION.

SPECIFIC OBJECTIVES

- State Newton's first law.
- Describe some applications of Newton's first law relate inertia to mass.
- State Newton's second law.
- Derive F = ma.
- Define the Newton as a unit of force.
- Calculate the weight of a body in a lift moving with constant velocity and under uniform acceleration.
- State Newton's third law.
- Solve simple numerical problems.

Force and Motion

As we have seen in chapter five that one of the effects of force when it acts on a body is that:

(i) Make a stationary object to move.
(ii) Increase the speed of a moving object.
(iii) Decrease or slow down the speed of a moving object or bring a moving object to a rest.
(iv) Change the direction of a moving object.
(v) Deform (change the shape of) an object.

The relationship between force and motion was stated by Galileo Galilli, an Italian scientist and died before he could complete his investigations. His works were continued and completed by an English Scientist called Sir Isaac Newton.

Newton carried out series of experiments and through experimental results; he summed up the basic principles underlining motion in three laws. These laws are known as Newton's Laws of Motion.

Newton's First Law of Motion.
Newton's first law of motion states that:
Everybody continuous in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Common experiences have shown that objects at rest do **not** begin to move on their own accord or objects in motion do **not** come to rest instantly on their own. As a result of this, the following are cited as examples of Newton's first law of motion.

(a) A body at rest

When a pile of coins is placed on a table, the one at the bottom can be removed by applying a sudden force to it without disturbing the ones on the top.

Explanation:

The force applied only acts on that particular coin at the bottom. Since the rest are not acted upon by the force, they remain undisturbed.

Note: If a slow acting force is applied to the lower coin, the whole pile od coins will move with it.

(b) A body on motion

(i) A person riding a bicycle along a level road does not come to a rest immediately when he stops pedaling. The bicycle continues to move forward for some time, but eventually comes to a rest.

Explanation

The bicycle continues to move because of inertia and comes to a rest after some time as a result of the retarding action of the external forces such as air resistance and frictional force between the tyre and the road surface. These external forces oppose the motion and the cyclist eventually come to a rest.

(ii) **Collision of two vehicles or when brakes are suddenly applied to a car moving at a high velocity.**

In the above incidences, passengers who do not fasten their safety belts are often injured when they jerk forward and hit the wind screen.

Explanation

An external force acts on the vehicle but not on the passenger who simply continue with their motion in a straight line in accordance with Newton's first law of motion.

(iii) **A bullet fired at an angle from a gun**.

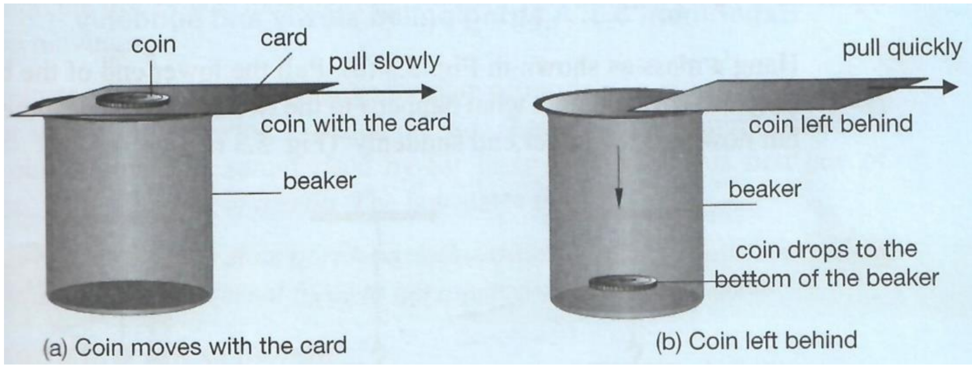
When a bullet is fired from a gun held at angle to the ground, the bullet travels and eventually falls to the ground.

Explanation

The motion of the bullet is opposed by air resistance and the gravitational force,hence, sooner or later it returns to the ground.

Note: As per Newton's first law of motion, it is supposed that, if the external forces such as friction between solid surfaces in contact, air resistance and gravitational force in the above examples were not there, the bodies would continue to move forever.

An experiment to demonstrate Newton's law.



A coin is placed on a smooth card board and then placed over the beaker.
The card is pulled away slowly as shown in (a)

Observation

The coin moves together with the card.

Explanation

The friction force between the coin and the card makes them to move together.
The experiment is repeated but this time the card is pulled away suddenly as shown in (b).

Observation

The coin is left behind and suddenly drops vertically into the beaker.

Explanation

When the card is moved suddenly the coin resists the motion and does not move with the card and hence drops vertically in the beaker. The coin resists changing its state of rest but due to lack of support from below falls into the beaker.

Inertia

Definition: **Inertia** is the tendency of a body to remain at rest or, if moving to continue its motion in a straight line.
Or
Inertia is the reluctance of a body to start moving or to stop moving once it has started.

For this reason, Newton's first law is sometimes called "the law of inertia".

A body of large mass requires a large force to change its speed or its direction i.e. the body has a large inertia. Thus, the mass of a body is a measure of its inertia.

Attempt Exercise 5.1 on page 76.

Newton's Second Law of Motion

Newton's second law explains the relationship between force applied to a body and the change in the momentum of the body that it causes.

Newton's First Law of Motion States that**:**

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's second law enables us to define the unit of force and establishes the fundamental equation of dynamics, **F = ma**.

Derivation of the formula
Suppose a force, **F,** acts on a body of mass, **m,** for time **t** and causes its initial velocity, **u,** to change to final velocity, **v**. The momentum changes uniformly from **mu** to **mv** in the time interval, **t**.

The rate of change of momentum $= \frac{\text{Final momentum - Initial momentum}}{\text{Time taken}}$

$$= \frac{mv - mu}{t}$$

By Newton's second law, the rate of change of momentum is proportional to the applied force and hence, $F \; \alpha \; \frac{mv - mu}{t} \Rightarrow F \; \alpha \; \frac{m(v - u)}{t}$ But $\frac{v - u}{t} = a$

$\therefore$ **F α ma**
Introducing a constant to change the sign of proportionality to equal sign, we have:
F = Constant × ma

If **m** = 1 kg and **a** = 1 ms^{-2}, the value of the unit of force is chosen so as to make **F** = 1. This implies that the value of the constant = 1.

$$\therefore \boldsymbol{F = ma}.$$

The SI unit of force is called the **newton** (symbol, N) and is defined as:

The force which produces an acceleration of 1 ms^{-2} when it acts on a mass of 1 kg.
Or
A force which give 1 kg mass an acceleration of 1 ms^{-2}.

Thus, when **F** is in Newton, **m** in kilograms and **a** in metres per second squared, we have
F = ma

Worked Examples

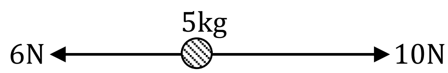
1. A force 3 N acts on a body of mass 5 kg. Find the acceleration produced.
   Solution:
   Mass = 5 kg, F = 3 N, a =?
   $F = ma \Rightarrow a = \frac{F}{m} = \frac{3}{5}$ **= 0.6 ms^-2^**
2. Find the force acting on a body of mass 12 kg and making it to produce an acceleration of 6 ms^{-2}.
   Solution
   m = 12 kg, F = ?, a = 6 ms^{-2}
   F = ma = 12 × 6 **= 72 N**

3. A resultant force of 6N acts on a body of mass 2 kg. What is the acceleration of the body?

Solution
Net force (F) = Mass(m) x acceleration(a)

$$\therefore \qquad a = \frac{F}{m} = \frac{6}{2} = 3\,ms^{-2}$$

4. Two forces 10N and 6N act on a particle of mass 5 kg as shown.

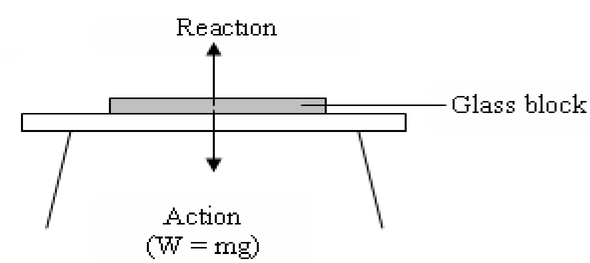


Find the acceleration of the particle.

Solution
Since the forces act along the same line in opposite directions, the net force on the particle is (10 – 6) = 4N in the direction of the bigger force.
Acceleration, a = F/m = 4/5 = 0.8 ms^{-2}

Attempt Exercise 5.2 on pages 81-82
Newton's Third Law of Motion:
Newton' s third of motion suggests that forces act in equal but opposite pairs for a body to be in a state of equilibrium.

Newton's third law of motion states that**:**
Whenever a force acts on one body, an equal and opposite force acts on some other body.
OR
To every action there is an equal and opposite reaction.



For example, a glass block placed on a table, exerts a force equal to its weight on to the table top. This force is called action. At the same time the table top exerts an equal force on to the glass block in the opposite direction. This force acting in the opposite direction is called reaction.
Note: - Action = Reaction
- Action and reaction act on different bodies.
- The two forces are in opposite directions.
- The net resultant force on the glass block is zero

Examples of Newton's third law include:
(i) A man jumping from a boat
A man jumping from a boat exerts action force on the boat and the boat exerts a reaction force. As he jumps to the river bank, the boat moves backwards.

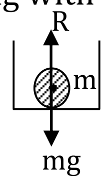
(ii) Propulsion of a bullet from a gun

When a bullet is fired from a gun, the energy of the explosion of the charge in the cartridge acts on both the bullet and the gun, thus producing equal and opposite forces acting on them. These equal forces act for the same time i.e the time taken by the bullet to travel up the barrel of the gum. The time effect of a force is called **impulse**; thus the bullet and the gun are given equal and opposite impulses. In each case, the impulse is equal to the change in momentum.

Attempt Exercise 5.4 on pages 88-89

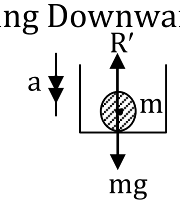
Weight of a body in a Lift

(a) Lift at rest or moving with constant velocity



When a body of mass m is placed on the floor of a lift, the body experiences a reaction R from the floor.
If the lift is stationary, or not accelerating, the net force on the body is zero. In this case the normal reaction, R is just equal to the weight, mg of the body

(b) Lift Accelerating Downwards



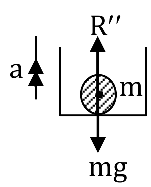
In this case there is a net downward force. So the reaction R′, experienced on the floor, is less than the weight, mg. R′ is the apparent weight of the body.
Now, the net force = mass × acceleration

$\therefore$ $mg - R' = ma$

$\therefore$ $R' = mg - ma = m(g - a)$

If the lift is left to **fall freely**, then a = g and R′ = 0. When this occurs, the body experiences weightlessness i.e. its net weight is zero.

(c) Lift Accelerating Upwards



This time there is a net upward force. So, the reaction, R″ is greater than the weight, mg, i.e. the body appears to weigh more.
Net force = mass × acceleration

$\therefore$ $R'' - mg = ma$

$\therefore$ $R'' = mg + ma = m(g + a)$

Example

A lift moves up and then down with an acceleration of 3 ms^{-2}. Calculate the reaction by the floor on a passenger of mass 60kg standing in the lift in each case.
Upward (780N)
Downward (420N)

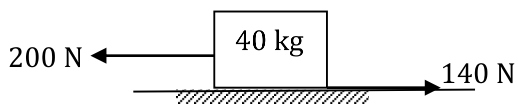
Attempt Revision Exercise 5 on pages 91-93

Test Yourself

1. A force of 100 N acts on a body and produces an acceleration of 2 ms^{-2}. What is the mass of the body?

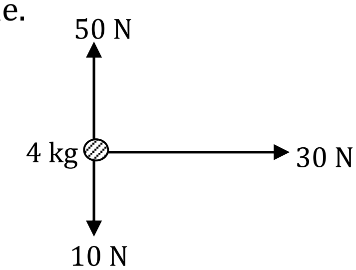
2. A car of mass 1200 kg moving with a velocity of 50 m s^{-1} is retarded uniformly to rest in 10 s. What is the retarding force?

3. A block of mass 40 kg is pulled from rest along a horizontal surface by a rope connected to one face of the block as shown below.



Given that the tension is 200 N and that the frictional force between the block and the horizontal surface is140 N, find
(i) the acceleration of the block.
(ii) the distance moved in 5.0 s

4. A particle of mass 4 kg is acted on by a system of forces as shown below. Find the acceleration of the particle.



5. A cylinder contains 10 kg of compressed gas. The valve is opened and after 20 s the mass of gas remaining in the cylinder is 4 kg. if the gas flows out of the nozzle at an average speed of 25 m s^{-1}, find the average force exerted on the cylinder.

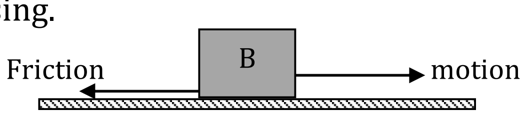
6. A helicopter wit crew and passengers rises with vertical acceleration of 5 ms^{-2}. The total mass of crew and passengers is 720 kg. Calculate the reaction exerted by the crew and passengers on the helicopter floor.

7. A spring balance carrying a mass of 4.0 kg on its hooker is hanged from the ceiling of a lift. Determine the spring balance reading when the lift is
(a) ascending with an acceleration of 0.4 ms^{-2}.
(b) descending with an acceleration of 4 ms^{-2}.
(c) ascending with a uniform velocity of 4 ms^{-1}.

SUB-TOPIC: FRICTION BETWEEN SOLIDS
SPECIFIC OBJECTIVES:
- Define friction.
- Carryout experiments to determine factors affecting static and dynamic frication.
- Compare static and dynamic friction.
- Describe advantages and disadvantages of friction.
- Give examples where consideration of friction is necessary.
- Describe methods of reducing or increasing friction.

FRICTION BETWEEN SOLIDS

Definition:
Friction is the force that opposes the relative sliding motion between two surfaces in contact with one another.
Therefore, friction is a vector quantity and it acts in the opposite direction of the motion it is opposing.



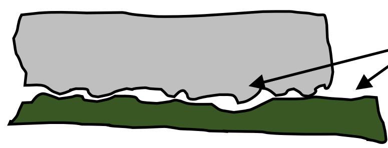
SI unit of friction is the **newton** (N).

Importance of friction (**Advantages of Friction)**
Friction makes it possible for:
- one to write.
- one to walk.
- a moving vehicle to brake,
- knife to be sharpened.
- a matchstick to produce fire.

Negative effects of friction (**Disadvantages of Friction)**
Friction:
- wears out surfaces.
- causes unnecessary noise.
- produces unnecessary heat.
- slows motion.
- reduces the efficiency of some machines.

The negative effects of friction can be overcome by:
(i) lubrication of the surfaces in contact.
Methods of lubrication include: oiling, greasing and air lubrication.
(ii) using smooth surfaces.
(iii) using rollers or ball bearings.

Origin of friction
(i) Roughness of materials
Most surfaces, however smooth they may appear, have ridges and bumps that are at times invisible to the eye. These ridges and bumps interlock when the two surfaces come into contact with each other and prevent movement of the two surfaces.



When a sufficient force is applied, it overcomes the interlocking forces between the ridges and bumps and makes movement possible.
The rougher the surfaces, the greater the friction. Therefore, frictional force depends on the **nature of the material** but not on the area of contact.

(ii) Attractive forces between the molecules of the surfaces.
This attraction between surface molecules may play a bigger role than roughness of the surfaces.

Factors that affect friction in solids

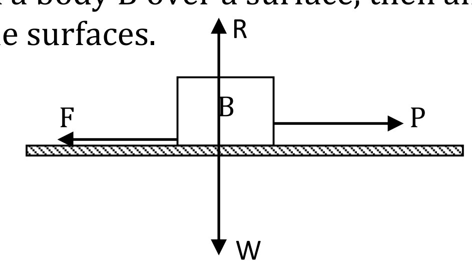
(i) **Area :** this does not affect friction. Friction is independent of area.
(ii) Normal reaction: Friction is directly proportional to normal reaction.
(iii) Nature of the surface in contact: The smoother the surface the smaller the friction force.

Types of friction:

There are two types of friction, namely;

1. Static (limiting) friction.
2. Dynamic (kinetic) friction.

When a force P is applied to pull a body B over a surface, then an equal opposing force (friction), F comes into existence between the surfaces.



As the force P is increased, the frictional force F increases equally. However, eventually the frictional force reaches a maximum value. Any increase in the pull P now moves the block. The maximum frictional force reached is called the **limiting frictional force** or **static friction** for the setup.
Once the block starts to slide, the pulling force, P reduces slightly and the friction between the block and surface is referred to as **dynamic** or **kinetic** friction.

Definitions:

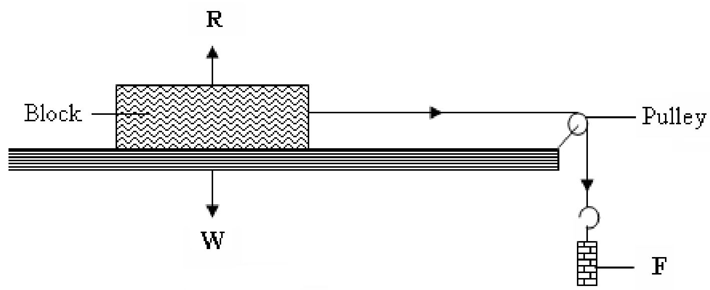
1. **Static or limiting friction is the maximum friction between any two surfaces that are in contact just before they start sliding over each other.**

2. **Dynamic or kinetic friction is the friction between two surfaces that are in contact when they are sliding over each other.**

The dynamic friction force is always less than the static friction.

Experiment: To Measure Static friction

Static friction can be measured using the apparatus shown in figure below



A wooden block is placed on a table and it is connected to one end of a light string passing over a smooth pulley with its other end attached to a mass hanger.
Force, **F**, is gradually increased by first adding 50 g and when the block is about to slide, smaller standard (known) 20 g or 10 g are continuously added on to the mass hanger until the block just begins to slide or move.

Force, **F,** is read and recorded (the total mass on the mass hanger) at the point when the block just begins to slide.

Observations

At first the block remained at rest as the force was increased.
After some time, i.e. at certain value of the force on the pan, the block just begun to slip or slide in the direction shown on the diagram.

Explanation

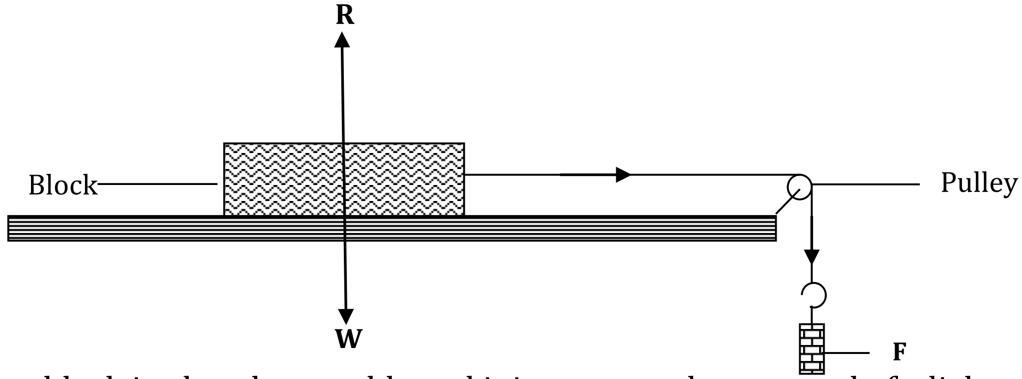
When force, **F,** on the mass hanger was increased the frictional force that opposes the motion of the block also increases.
As more and more weights were added to the mass hanger, the frictional force reached its maximum value for the two surfaces in contact and begun to slide.

Result

The maximum value of the frictional force is equal to the total weight, **F**, on the mass hanger. This maximum frictional force is called static or limiting friction.

Experiment: To Measure sliding friction

Sliding or dynamic ors kinetic friction can be measured using the apparatus shown in figure below.



A wooden block is placed on a table and it is connected to one end of a light string passing over a smooth pulley with its other end attached to a mass hanger.

Force, **F**, is gradually increased by first adding 50 g or 20 g or 10 g on to the mass hanger until the block is about to slide.

The block is given a push.

Force, **F,** is read and recorded (the total mass on the mass hanger).

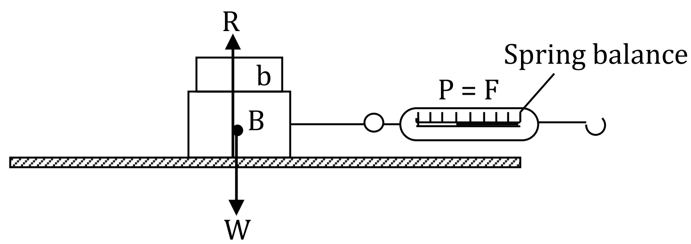
Result

The maximum value of the frictional force is equal to the total weight, **F**, on the mass hanger. This maximum frictional force is called sliding or kinetic friction, $\mathbf{F}_k$.

Relationship between friction and the normal reaction.

Friction bears a relationship to the normal reaction offered by the supporting surface to the block. If the surface is horizontal, the normal reaction is equal to the weight of the block.

Experiment: To Investigate the Relationship between the Limiting Frictional Force and the Normal Reaction



- A block B is weighed and placed on a horizontal surface
- The block is then pulled with a horizontal spring balance and when it is at the point of beginning to slide, the reading, P, on the spring balance is noted.
- A weight b is added to the block. The total weight, W, of the B plus what is added, is found and B is pulled to find the new value of P.
- The procedure is repeated for several other weights added and the results are recorded in a table as shown below.

Total weight, W (N)	P (N)	$\frac{P}{W}$

The ratio $\frac{P}{W}$ is found to be constant.

Now, P is equal to the limiting frictional force, F, and W is equal to the normal reaction, R. So, the limiting frictional force, F is directly proportional to the normal reaction, R. The ratio, $F/_R$ is known as the **coefficient of static friction** between the two surfaces.

$$\therefore F = \mu R$$

Alternatively, plot a graph of P against W and find its slope. The slope is equal to the coefficient of friction.

Experimental results show that friction increases with the force pressing the two surfaces together, known as the normal reaction.

The coefficient of friction is a constant for any given pair of surfaces. Therefore,

coefficient of friction, $\mu = \frac{F_1}{R_1} = \frac{F_2}{R_2}$

Laws of friction.

The laws of friction explain the behaviour of friction. They are:

1. Friction opposes relative motion of surfaces in contact.
2. The maximum (limiting) friction force is directly proportional to the normal reaction but independent of the area of contact.
3. The kinetic frictional force is independent of the relative speed between the two surfaces but dependent on the normal reaction.
4. Friction increases with roughness of the two surfaces in contact.

Example

A box of weight 20N rests on a horizontal floor. A minimum horizontal force of 6N is required to move the box along the floor. If a weight of 10N is added to the box, find the minimum horizontal force required to move the block.

Solution

The minimum force required is the limiting frictional force.
Let F be the required force in the second case.
Then,

coefficient of friction, $\mu = \frac{F_1}{R_1} = \frac{F_2}{R_2}$

$\frac{6}{20} = \frac{F_2}{20 + 10}$

$F_2 = 9 \text{ N}$

Example 1

A block of wood of mass 5 kg is placed on a table top. Find the limiting friction if the coefficient of friction is 0.5. (Take g = 10 ms^{-2}).

Solution

Data: μ = 0.5, m = 5 kg, g = 10 m/s^2, Limiting friction, F = ?, R = ?
Hint: Limiting friction is calculated from the formula F **= μR.** But R is not given in the data. This should make the whole thing difficult to you.
Always remember that R = W (Newton's third law). So using the formula W = mg, find the value of W and then finally use the formula **F = μR** to get F as shown below.

First calculate W: W = mg
= 5 x 10
= 50 N
From Weight = Reaction ∴ R = 50 N
Now that you have known R, you can now use the formula;
Limiting friction, F = μR
= 0.5 x 50
∴ F = 25 N

Self-check

Try to check your understanding by answering the following questions.

1. A chalk box of mass 400 g is placed on a table. Find the limiting friction if $\mu = 0.2$, ($g = 10\ ms^{-2}$). **Answer: 0.8 N**

2. A block of wood of mass 2 kg is placed on a table. Find the limiting friction if $\mu = 0.4$, ($g = 10\ ms^{-2}$). **Answer: 8 N**

Attempt Revision exercise 6 on page 98 in Longhorn Book three.

SUB-TOPIC: MECHANICAL ENERGY

SPECIFIC OBJECTIVES

- Define P.E and K.E
- Derive formulae for K.E and P.E.
- Solve numerical problems.

In mechanics, mechanical energy is divided into two kinds, namely:

1. Potential energy and
2. Kinetic energy.

a) Potential energy (P.E)

Potential energy is the form of energy possessed by a body as a result of its position at rest or state.

For example, a body lifted to a height, h, above the surface of the earth is said to possess p.e as a result of its position above the earth.

When something is lifted vertically upwards, work is done against the gravitational force acting on the body (i.e. its weight) and this work is stored in the body as gravitational potential energy.

Another example of Potential energy is the elastic potential energy stored in a stretched spring or catapult.

Formula of Potential Energy

Suppose a body of mass **m** kg is raised to a height of **h** metres at a place where the acceleration due to gravity is **g** ms^{-2}.

Then Force overcome (Weight), F = mg …………………(1)

Work done on the body = F x h …………………(2)

Substituting equation (1) in equation (2) i.e replacing F in equation (2) with **mg** in equation (1) we have,

Work done on the body = mg x h

But the work done = gravitational potential energy (P.E).

∴ **P.E = mgh**

Recall that: m = mass of the body in kg.

g = acceleration due to gravity (ms^{-2}).

h = height in metres.

Examples

1. A box of mass 5 kg is raised to a height of 2 metres above the ground. Calculate the potential energy stored in the box (take g = 10 ms^{-2})

Solution:

Mass of box = 5 kg, gravitational field, g = 10 ms^{-2} Height, h = 2 m,

Applying P.e = mgh

= 5 x 10 x 2

∴ **P.E = 100 J or 0.1 kJ**

2. A man has raised a load of 25 kg on a platform 160 cm vertically above the ground. If the value of gravity is 10 ms^{-2}, calculate the potential energy gained by the box when it is on the platform.

Solution:

Mass of stone = 25 kg, gravitational field, g = 10 ms^{-2}

Height, h = 160 cm = $\frac{160}{100}$ = 1.6 m, P.E = ?

P.E = mgh

= 25 x 10 x 1.6

∴ **P.E = 400 J or 0.4 kJ**

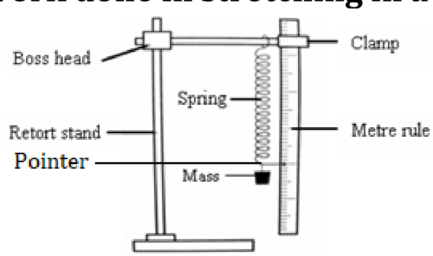
3. A ball is dropped from rest at a height of 20 m above the ground. If the ball bounces on hitting the ground and lost 20% of its original energy, calculate the maximum height it reaches again.

Elastic potential energy.

This is potential energy possessed by bodies by reason of their elastic or compressed state.

A compressed or stretched spring goes back to its original shape when released. The work done on stretching or compressing the spring is called elastic potential energy.

Experiment: To determine the work done in stretching in a spiral.



The original length of the spring is measured when no mass is hung from the free end.

This length is recorded as, l_0.

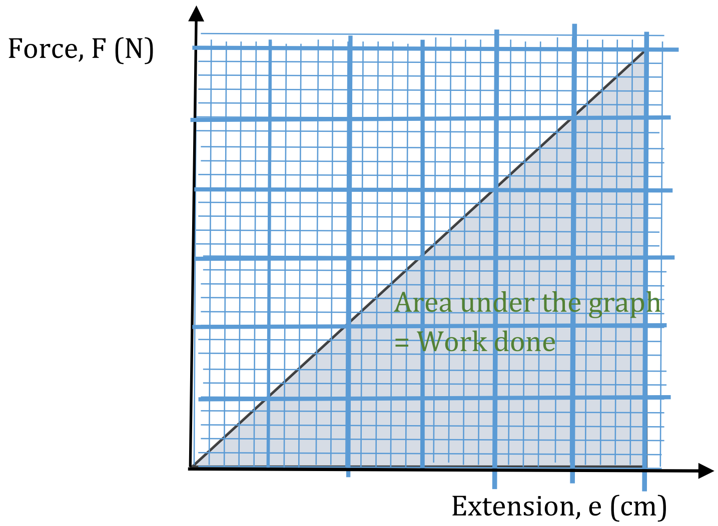
A mass is hung from the spring and the new length, l_1.is measured.

The difference, e between the new length, l_1 and the original length, l_0 is found. e is the extension of the spring.

The experiment is repeated using other masses and the results are record in a table as shown below.

Mass (g)	Weight (N)	Extension, e (cm)

A graph of force, F against extension, e is plotted.



The area under the graph represents the work done in stretching the spring. This work is stored in the spring as elastic potential energy.

work done = average force apllied × extension

$$\text{work done} = \frac{0 + F}{2} \times e = \frac{1}{2}Fe$$

work done = area under the graph.

From Hooke's law, applied force = spring constant × extension.

$$F = ke$$

Substituting in the previous equation of work,

$$\text{work done} = \frac{1}{2}ke^2 .$$

Example.

How much work is done in stretching a spring of spring constant 25 Nm^{-1} when the length is increased from 0.10 to 0.20 m?

Solution:

$$\text{work} = \frac{1}{2}ke^2 = \frac{1}{2} \times 25 \times (0.2 - 0.1)^2 = \frac{1}{2} \times 25 \times 0.1^2 = 1.25 \times 10^{-1}\ \text{J}$$

(b) Kinetic energy (K.E)

Kinetic energy is the energy possessed by a moving body.

Examples of kinetic energy include:

- Moving bullet, Moving car, etc.

A body possessing k.e does work by overcoming resistance force when it strikes something.

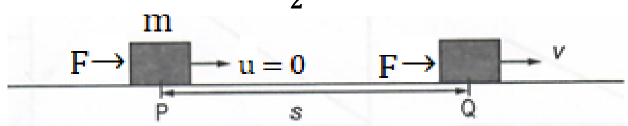
Formula of Kinetic energy (K.E)

Kinetic energy can be calculated from the formula;

$\mathbf{K.E = \frac{1}{2}mv^2}$ Where, m = mass (kg), v = velocity (ms^{-1}) .

Note that: Kinetic energy is directly proportional to the speed or velocity of a body. Therefore, the faster the body moves, the more the kinetic energy it has.

The derivation of the above formula, K. E = $\frac{1}{2}$mv^2.



When a body of mass, **m, initially at rest** and is acted upon by a force, **F**, the force gives the body an acceleration, **a,** and its velocity increases to a final velocity, **v,** after covering a distance, **s** in metres.

These quantities are related by the equation of linear motion (See Eqns of uniformly accelerated motions).

$$\mathbf{v^2 = u^2 + 2as}$$

Since u = 0 then $v^2 = 2as \Rightarrow a = \frac{v^2}{2s}$ ………………… (1)

The work done on a body in moving from Point P to point Q is the kinetic energy of the body at point B.

Now, Work done = Force x Distance = F x s

But F = ma (Newton's second law of motion)

∴ Substituting for F, we have:

Work done = mas ……………………………(2) s represents displacement.

Substituting equation (1) in equation (2), i.e. substituting for **a** in equation (2), we obtain

Work done $= \frac{mv^2 s}{2s}$

Work done $= \frac{1}{2}\mathbf{mv^2}$

But gain in Kinetic energy = work done

$$\mathbf{K.E = \frac{1}{2}mv^2}$$

Worked Examples

1. Calculate the K.E of a bullet of mass 0.05 kg moving with velocity of 500 ms^{-1}.

   Solution: m = 0.05 kg, v = 500 ms^{-1}, K.E = ?

   Kinetic Energy = ½mv^2 = ½ × 0.05 × 5002

   $= \mathbf{6,250\ J\ \ or\ \ 6.25\ kJ}$

2. A 10 g bullet traveling at 400 ms^{-1} penetrates 20 cm into a wooden block. Calculate the average force exerted by the bullet.

   Solution: m = 10 g = $\frac{10}{1000}$ kg, v = 400 ms^{-1}, distance = 20 cm = $\frac{20}{100}$ m, K.E = ?

   Hint: The work done in penetrating the block is related to the average force by the formula:
   Work Done = Fs, so find the work done first and then use the above formula to find F.

   Kinetic Energy $= \frac{1}{2}$mv^2 $= \frac{1}{2} \times \frac{10}{1000} \times 400^2 = 5 \times 4 \times 40 = \mathbf{800\ J}$

   This kinetic energy is converted into work in penetrating the wooden block

   But Work done = Force × Distance

$800 = F \times \frac{20}{100}$

$20\,F = 800 \times 100$

$F = \frac{80000}{20}$

$\mathbf{F = 4000\ N}$

3. A particle of mass 2 kg is at rest, freely suspended on a string. It is then struck horizontally and starts off with a velocity of 10 ms^{-1}.
   (a) Find how high above the initial position is rises.
   (b) What kinetic energy does it have on returning to the initial position?

   Solution:

   (a) Let h = height risen
   Potential energy gained = kinetic energy lost

   $$\therefore\ mgh = \frac{1}{2}mv^2$$

   $$\therefore\ h = \frac{v^2}{2g} = \frac{10^2}{2\times 10} = 5\,m$$

   (b) When the particle returns to the point of projection it will have the same kinetic energy as it had when it was leaving equal to $\frac{1}{2}mv^2$

   $$= \frac{1}{2} \times 2 \times 10^2 = 100J$$

4. A body of mass 3.0kg starts from rest. Find its kinetic energy after travelling through a distance of 5m with a uniform acceleration of 2 ms^{-2}. (30J)

5. A body of mass 12kg is pulled from rest with a constant force of 25N. The force is applied for 6.0s. Calculate;
   (a) the distance travelled (37.5m)
   (b) the work done on the body (945J)
   (c) the final kinetic energy (945J)
   (d) the final velocity of the body (12.5 ms^{-1})

Attempt Revision Exercise 7 on pages 103 -105 in Longhorn Book three.

THE END.